How a Golf Swing Carries Energy

A Book-Length Visual Journey Through Motion, Forces, Timing, Models, and Uncertainty

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This is a patient exploration of a moving system. It follows energy from the ground, through the body and hands, into a flexible club, and toward the ball. Ordinary scenes make the mechanics visible; equations and experiments keep the scenes honest. No model result becomes swing instruction merely because it is easy to picture. The purpose is to make the scientific questions understandable enough that a reader can examine, challenge, and improve them.

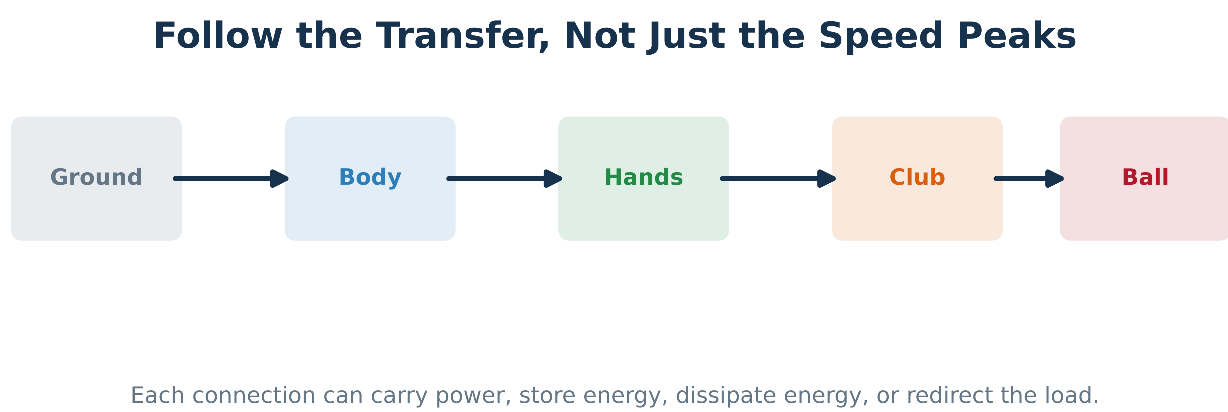

Figure 1: A Five-Stage Map Follows Mechanical Energy From the Ground Through the Body and Hands to the Club and Ball.

How to Read This Book

The book is organized as a long walk rather than a stack of definitions. Part I builds the language of motion. Part II follows forces through a double pendulum. Part III opens the two-hand and flexible-shaft mechanisms. Part IV shows how models grow without pretending that the largest model is automatically the best. Part V asks what is identifiable, robust, measured, and falsifiable. The final part turns the argument into a reviewer path and a neutral practical synthesis.

Every chapter has four recurring landmarks:

- **A Concrete Picture** begins with an ordinary situation that can be held in the mind while the mechanics develops.

- **How the Mechanism Works** supplies the sustained explanation, equations, figures, and worked reasoning.
- **Where the Picture Breaks** states exactly where the analogy or reduced model stops earning conclusions.
- **Go Deeper** links to the corresponding technical chapter, executable evidence, and real literature.

Claim labels recur throughout. **Model Result** means an executable mathematical system produced a result under declared assumptions. **Human Evidence** means people were measured. **Hypothesis** means a proposed explanation awaits a decisive test. **Practical Interpretation** translates a question into ordinary language without declaring a universal technique.

Readers who want to change the models can use the interactive proximal-distal workbench. Readers who want every equation and numerical contract can open the complete scientific monograph.

This book is **not a coaching instruction**. It explains testable mechanics, evidence boundaries, and open questions. The preload discussion also links to the machine-readable transmission robustness study used by the technical analysis.

Chapter 1

Follow the Energy

1.1 A Concrete Picture

Imagine standing above a busy railway yard at dusk. A locomotive enters with fuel in its tanks and motion in its wheels. It couples to a line of cars. Some cars begin moving, one spring-loaded coupling compresses, brakes warm on another car, and the last car eventually rolls faster than the first. If we watched only the peak speed of each car, we would miss nearly everything that made the event interesting. We would not know which engine supplied work, which coupling stored energy for a moment, which brake removed it, or which force merely kept a car on the curved track.

A golf swing is smaller and faster, but the accounting problem is similar. The pelvis, trunk, arms, hands, shaft, and clubhead do not behave as a row of rigid blocks passing a package along a conveyor. They are linked bodies whose geometry changes while forces act. A connection can transmit power in either direction. A segment can slow while still doing positive work on its neighbor. A large force can redirect motion while transferring almost no energy. A flexible shaft can accept energy, keep it briefly, and return some later. Gravity can help one part of the path and oppose another. Damping quietly turns organized mechanical energy into heat.

The first discipline of this book is therefore simple to state and surprisingly hard to maintain: follow the complete energy ledger, not just the visible order of speed peaks.

The familiar phrase *proximal to distal* means, roughly, from nearer the center of the body toward farther-away segments. In golf, researchers often describe a sequence in which pelvis speed peaks, then trunk speed, then arm or hand speed, and finally club speed. That ordering is real enough to be measured and useful enough to study (Putnam 1993; Tinmark et al. 2010). But it is an observation about motion. It is not yet an explanation of energy flow. The explanation requires forces, moments, velocities, and the boundaries across which power is computed.

1.2 How the Mechanism Works

Begin with a modest definition. Mechanical energy is a bookkeeping quantity for a chosen system. A moving rigid body has translational and rotational kinetic energy. A raised body has gravitational potential energy. A bent shaft or stretched tendon may have elastic potential energy. When external forces do work, the system's mechanical energy can change. When internal forces act between parts of

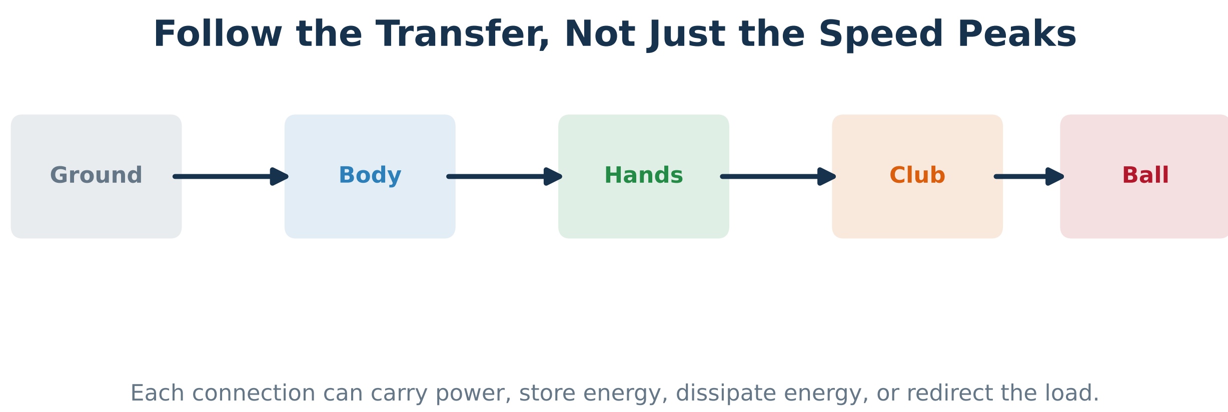

Figure 1.1: A Five-Stage Map Follows Mechanical Energy From the Ground Through the Body and Hands to the Club and Ball.

a larger chosen system, they redistribute energy among those parts even though they do not create energy for the whole system.

That last sentence is the heart of the matter. Suppose we draw one boundary around the golfer and club together. The forces between a hand and the grip are internal to that combined system. Their positive work on the club is paired with an equal-and-opposite interaction on the hands, although the two powers need not cancel if the contact points and compliant elements are represented differently. Now draw a smaller boundary around the club alone. The same hand forces become external inputs to the club. They can change its kinetic energy, bend its shaft, or redirect its motion. Neither boundary is more correct. They answer different questions.

For a force $\mathbf{F}$ acting at a point moving with velocity $\mathbf{v}$, the instantaneous power delivered through that point is

$$P_F = \mathbf{F} \cdot \mathbf{v}.$$

For a moment τ acting on a body with angular velocity ω, rotational power is

$$P_\tau = \tau \cdot \omega.$$

These dot products are more informative than force or torque magnitude alone. When force points generally with velocity, power is positive. When it points against velocity, power is negative. When it points perpendicular to velocity, it may bend the path strongly while doing no instantaneous work. A centripetal force in uniform circular motion is the clean textbook example: it is essential to the curved path, yet its dot product with tangential velocity is zero.

This gives us several distinct verbs for the swing. An actuator can **supply** energy by positive work.

An interaction can **transport** energy across a chosen boundary. A spring can **store** and later **return** energy. Damping can **dissipate** it. A perpendicular force can **redirect** motion. A changing constraint can **repartition** kinetic energy among coordinates. Saying only that energy "flows down the chain" compresses all those verbs into one and can hide the mechanism we most need to understand.

Consider a relay race, another useful picture. The order in which runners reach their top speed resembles a kinematic sequence. The baton resembles a physical object crossing a boundary. But a golf swing has no single baton of energy. Energy is a scalar assigned to states, and power is the rate at which work crosses a chosen boundary. What passes through the wrist is not a glowing substance. It is a calculable mechanical interaction whose sign and magnitude depend on the force, moment, motion, reference point, and frame.

The technical study uses an energy ledger precisely because several mechanisms can overlap. In a double-pendulum model, shoulder torque can add energy while the arm-club interaction redistributes it. In the two-hand model, a pair of opposed hand forces can create a club moment even when their net force is small. In the flexible-shaft model, interface power can move into elastic storage and return later. In a full-body constrained model, ground reactions enforce the motion and may transmit power through the feet. The ledger asks each contribution to identify itself rather than accepting one story for all of them.

1.2.1 A Small Numerical Ledger

Suppose a simplified club receives 500 watts of positive hand power for 0.10 seconds. If the power were constant, the hands would deliver 50 joules of work. During the same interval, imagine gravity contributes 3 joules, damping removes 5 joules, and the shaft finishes with 8 joules more elastic energy than it began. The remaining increase in rigid-body kinetic energy is

$$\Delta K = 50 + 3 - 5 - 8 = 40 \text{ J.}$$

Nothing here says the shaft "lost" eight joules. At the chosen final time, those joules are stored rather than expressed as rigid-body kinetic energy. If the shaft later straightens and returns six joules while dissipating two, the ledger still closes. Timing decides which account holds the energy near impact.

The same care is needed when a proximal segment slows. A slowing arm may be losing kinetic energy, but that loss is not automatically equal to the club's gain. The shoulder may still be doing work. Gravity and damping may contribute. Interaction power may move energy between arm and club. Their kinetic energies also depend on configuration because the inertia matrix of a linked system is not a set of independent constants. The only safe claim comes from a closed balance in a declared model or from measurements rich enough to estimate one.

Model Result: In the open proximal-distal experiments, work-energy residuals are checked under timestep refinement. This supports the internal consistency of those simulations; it does not prove that every biological pathway has been modeled.

Human Evidence: Skilled golfers often exhibit ordered segment-speed peaks, and joint-work studies show substantial work throughout the body. These measurements motivate the ledger, but peak order alone does not identify the route of transfer (Putnam 1993; Tinmark et al. 2010).

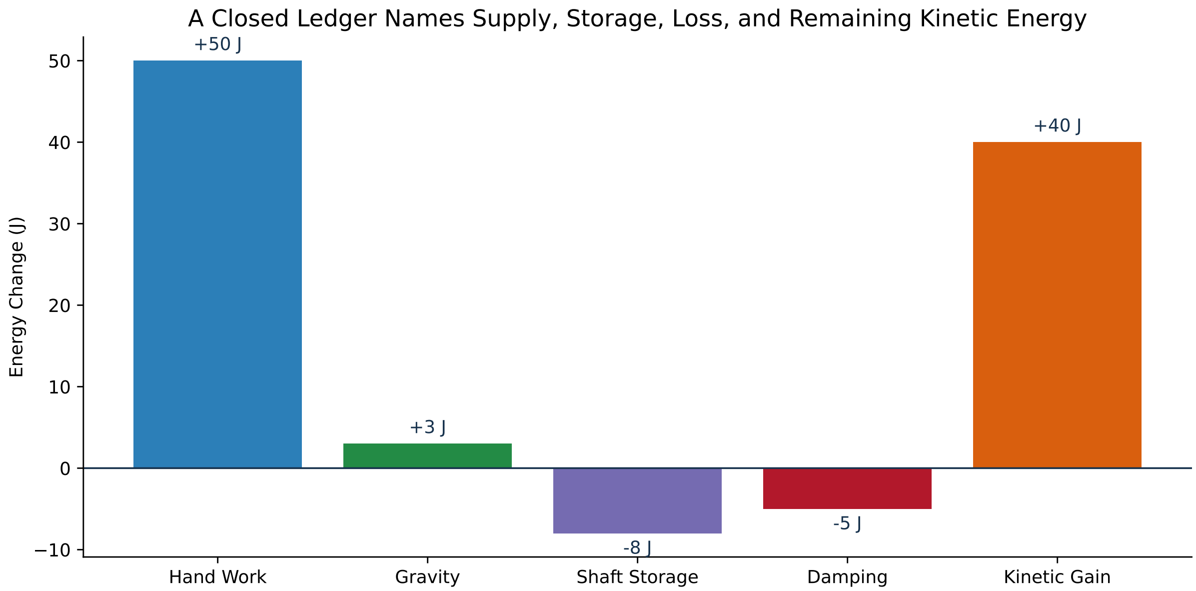

Figure 1.2: A Worked Energy Ledger Separates Hand Work, Gravity, Shaft Storage, Damping Loss, and the Remaining Kinetic-Energy Gain.

Hypothesis: A useful late-downswing strategy may depend on arranging the state so passive and interaction terms assist the club while direct distal actuation is delayed or even briefly opposing. The rest of the book makes this proposal precise enough to challenge.

Practical Interpretation: Instead of asking which body part "creates all the speed," ask what each boundary is doing during each phase: supplying, carrying, storing, returning, dissipating, or redirecting mechanical energy.

1.2.2 A Swing-Phase Walkthrough

At address, almost none of the mechanical story has happened. The club is held above the ground, so gravity has already given the golfer–club system potential energy, but the important fact is not the precise number. The important fact is that every later claim must name its ledger. During the backswing, muscles do work to raise and rotate body segments and the club. Some of that work becomes gravitational potential energy, some becomes kinetic energy, some is stored in deformed tissue or the shaft, and some is dissipated. Saying that the backswing "loads" the swing is therefore reasonable only after "load" is divided into measurable forms.

Around transition, club speed may be low while forces and internal activity are not. The direction of motion is changing, the golfer is reorganizing the state, and compliant elements may carry load. This is a useful antidote to the idea that low visible speed means nothing mechanical is happening. Early in the downswing, proximal work can increase the energy of the whole system while the club is still largely being carried. The club's own speed need not immediately mirror the rate at which energy enters elsewhere.

In the middle and late downswing, the ledger becomes especially interesting. The club can gain kinetic energy through power crossing the hands, through the return of shaft energy, and through coupling with the moving body. At the same time, one generalized torque can have negative power and still participate in a state evolution that produces greater later club speed. Near impact, energy in translation, rotation, and shaft deformation all affect the collision. After impact, substantial energy remains in the body and club or is dissipated; the ball never receives the entire pre-impact system energy.

1.2.3 A Numerical Thought Experiment

Suppose two simulated swings both deliver a clubhead at 45 metres per second. In Swing A, the club's effective moving mass for a chosen comparison is 0.20 kilograms; in Swing B it is 0.22 kilograms. The corresponding translational kinetic-energy estimates are about 203 and 223 joules. Equal speed has not meant equal energy. Now suppose Swing A reaches that speed with 400 watts crossing an interface for half a second, while Swing B has a 1,000-watt peak lasting only a few hundredths of a second. Peak power alone still does not say which boundary supplied more work. Integrating the complete histories does.

The numbers are illustrative, not measurements of a golfer. Their job is to force the vocabulary apart. Speed is a state variable, energy is a ledger quantity, power is a rate, and work is an accumulation. When a result is surprising, returning to those four definitions usually reveals whether the surprise is physical or merely verbal.

1.2.4 Questions for a Skeptical Reader

Before accepting any energy-transfer plot, ask whether all external powers were included; whether the sign convention is stated; whether the shaft was modeled as rigid or flexible; whether joint power was assigned consistently; and whether numerical energy residuals are small relative to the reported effect. Then ask the separate experimental questions: Were inertial parameters measured or assumed? Were force and motion synchronized? Does uncertainty in those inputs change the conclusion? A closed ledger earns the right to continue the argument. It does not finish it.

1.3 Where the Picture Breaks

Energy is not a liquid, electricity in a wire, traffic on a road, or a baton. Those images help us remember conservation and boundaries, but they can mislead us into imagining a material substance with a unique route. In a multibody system, the decomposition itself depends on coordinates and system boundaries. Some internal transfers can be described in more than one mathematically valid way. What must remain invariant is the whole-system balance and the observable motion, not every narrative label attached to an internal term.

The railway picture also makes couplings look sequential. In a swing, forces act throughout the system at the same time. The distal club can influence the proximal arms immediately through action-reaction. A shaft mode can affect grip loads before its stored energy is returned. The correct picture is a coupled network with time-varying geometry, not a one-way bucket brigade.

Finally, a closed simulated ledger is necessary but not sufficient. A wrong model can conserve its own energy perfectly. Numerical closure shows that the implemented equations agree with their accounting. Experimental agreement is a separate question requiring measured motion, wrenches, shaft deformation, ground reactions, and uncertainty.

Go Deeper: Read the scientific monograph's mechanics chapter, the detailed interaction-force decomposition, and Robertson and Winter's segment-energy framework (Robertson and Winter 1980). The open evidence directory contains the machine-readable ledgers used by the technical study.

Chapter 2

Choose the System Before Counting

2.1 A Concrete Picture

Picture a household checking its monthly finances. If the "system" is one person's wallet, a payment to a spouse is money leaving the system. If the system is the whole household, the same payment is an internal transfer: one account falls while another rises, but household wealth does not change. Rent paid to a landlord is external under either household boundary. A loan adds cash now and an obligation later. The accounting only becomes meaningful after we say what is inside the boundary and what quantities the ledger tracks.

Mechanical accounting works the same way. A statement such as "the arms gave energy to the club" quietly draws a boundary around the club. A statement such as "the golfer-club system gained energy from the ground" draws a larger boundary. Both can be useful. Trouble begins when numbers computed for one boundary are interpreted as though they belonged to another.

The choice is not philosophical decoration. It decides which forces are external, which interactions are internal, which energies appear in the ledger, and what conservation statement should close. Before counting transfer, we must choose the system.

2.2 How the Mechanism Works

Start with the club alone. Its rigid-body kinetic energy includes translation of its center of mass and rotation about that center. If the shaft is flexible, the club system also has elastic strain energy and possibly energy in several vibration modes. The hands exert external forces and moments on this chosen system. Gravity is external. Aerodynamic drag is external. If the club strikes the ball, contact with the ball is external. A club-only balance can therefore answer: how much mechanical work entered through the grip, how much gravity added, how much drag removed, how much remained stored in flex, and how much appeared as club kinetic energy?

Now enlarge the boundary to include the golfer and club. Hand-grip forces are internal action-reaction pairs. They still matter because they redistribute energy among body segments and the club, but they do not appear as net external work on the combined idealized system. Ground reactions, gravity, air drag, and metabolic-to-mechanical conversion remain relevant at or beyond the boundary. This

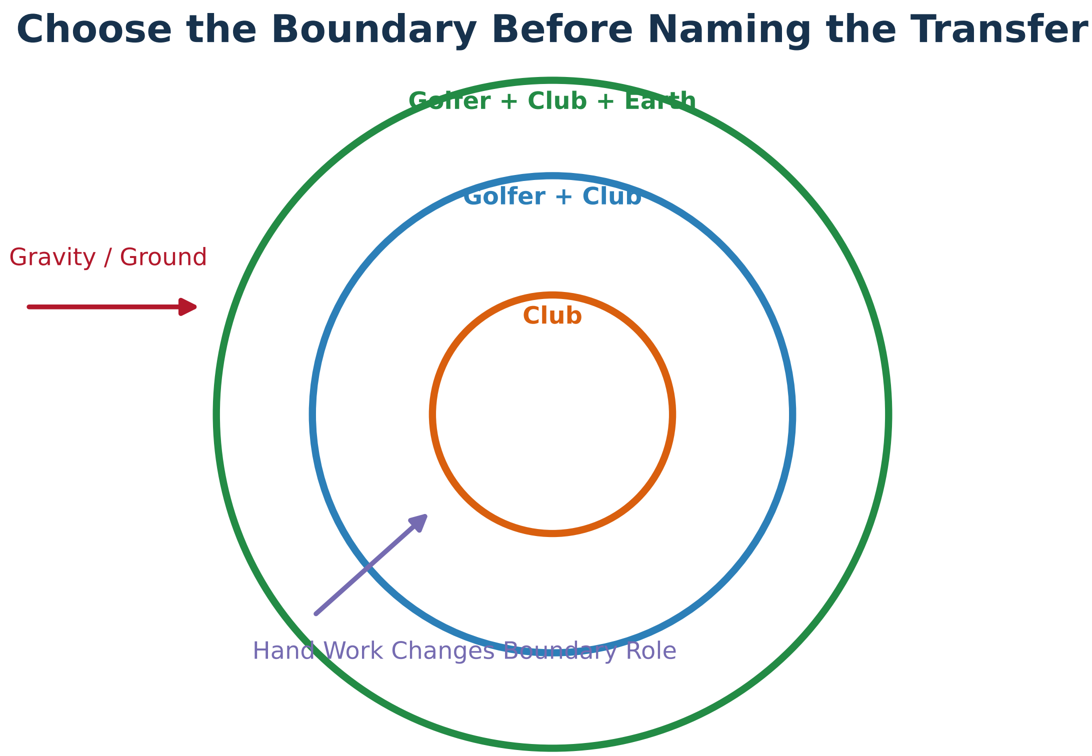

Figure 2.1: Nested Boundaries Separate Club, Golfer-Club, and Golfer-Club-Earth Energy Accounts.

larger balance can answer a different question: how did external work and internal actuation change the mechanical energy of the entire moving system?

Enlarge the boundary again to include Earth. Gravitational potential energy is now stored within the chosen system rather than supplied by an external gravity force. Ground contact becomes an internal interaction with Earth, although the Earth's change in kinetic energy is ordinarily negligible because its mass is enormous. The equations have not contradicted one another. The labels have changed because the boundary changed.

2.2.1 Internal Forces Can Still Transfer Energy Between Parts

Readers sometimes hear that internal forces "do no work." That phrase is only safe for particular idealized whole-system sums. An internal force can do positive work on one subsystem and negative work on another. Consider two skaters pushing apart. The contact forces are internal to the pair, but each skater gains kinetic energy because biological chemical energy is converted by muscle action during the push. If we examine one skater, the other's contact force is external and plainly does work.

For the arm and club, the wrist reaction force can carry power across the arm-club boundary. Let $\mathbf{F}_{H\to C}$ be the hand force on the club and $\mathbf{v}_H$ the velocity of the grip point on the club. The club-side power through that contact is $\mathbf{F}_{H\to C} \cdot \mathbf{v}_H$ plus any direct moment power. The equal and opposite club force on the hand participates in the body-side balance. With perfectly coincident rigid contact points, the pair's translational contact powers cancel in the combined-system sum. With compliance, relative motion, or a different partition, energy can temporarily enter stored interface states.

This is why words like *source*, *transfer*, and *storage* need care. Muscles are sources of mechanical work only by converting biochemical energy. The hand-club interface is usually a transfer boundary, not an independent energy source. A shaft spring stores energy but does not create it. Damping removes recoverable mechanical energy from the modeled degrees of freedom. Gravity exchanges kinetic and potential energy. Each verb belongs to a declared boundary and model.

2.2.2 Reference Points Matter for Moments and Power

A force is a vector that can be expressed in different coordinate frames without changing the physical interaction. A moment is always about a point. Move the reference point, and the force moment changes according to

$$\mathbf{M}_B = \mathbf{M}_A + (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}.$$

The combined force-moment pair is called a **wrench**. Its power is invariant when the force, moment, translational velocity, and angular velocity are all transformed consistently. But comparing a moment about the lead hand with a moment about the club center without performing the transformation can create a false difference. The two-hand chapters will return to this point because a pair of separated forces may have nearly zero resultant force yet a substantial moment about the club.

Reference levels also matter for potential energy. We may choose the ground as zero gravitational potential or choose the club's address height. Only changes in potential energy affect the balance, so either reference is valid if applied consistently. Absolute numbers without a declared zero are not comparable.

2.2.3 The Boundary Decides the Question, Not the Answer

Suppose a simulation reports that the club gains 40 joules during a phase, while the arm loses 12 joules. It would be wrong to conclude that exactly 12 joules were "transferred from arm to club" and the remaining 28 appeared mysteriously. Shoulder and wrist actuators may do work, gravity may contribute, the shaft may release stored energy, and other body segments may exchange power through the arm. A valid attribution needs interface powers and the complete ledger for the chosen partition.

Conversely, a whole-system balance can hide the internal route. If total mechanical energy rises by 50 joules and the numerical residual is tiny, we still do not know whether hip, shoulder, wrist, or elastic mechanisms delivered the work. Conservation is a consistency condition, not an anatomical diagnosis. To locate pathways, we deliberately draw subsystem boundaries and compute power through each interface.

Model Result: The technical work reports grip power, shaft-interface power, actuator work, gravitational work, damping loss, stored elastic energy, and whole-system residuals for multiple model tiers. Agreement of those ledgers is evidence that the implemented partitions are mechanically consistent.

Human Evidence: Inverse-dynamics studies can estimate net joint moments and powers from measured kinematics and external forces, but results depend on body segment parameters, filtering, joint definitions, and the treatment of two-hand contact. Net joint power is not a direct measurement of a particular muscle's work (Nesbit and Serrano 2005; Winter 2009).

Hypothesis: A boundary placed around the club and shaft may expose transfer pathways that are invisible in segment-speed plots, particularly when a negative grip couple coexists with positive power arriving through another channel.

Practical Interpretation: Whenever someone says energy was "generated," "stored," or "transferred," ask three quiet questions: what system was chosen, what crossed its boundary, and did the ledger close?

2.2.4 The Same Swing Under Four Boundaries

Imagine drawing four outlines on the same video frame. The smallest outline contains only the club. Every force and moment applied by the hands crosses that boundary, so hand power is external to this system. Shaft strain energy is internal if the flexible shaft remains inside the outline. Aerodynamic drag and the impact force are external. This boundary is excellent for asking how the club acquires and loses mechanical energy, but it says almost nothing about how the body produced the hand wrench.

The next outline contains both hands and the club. Grip contact becomes an internal exchange; wrist and forearm loads cross the new boundary. This view is useful for separating a two-hand internal couple from the net wrench delivered by the arms. The third outline adds the upper body. Shoulder, trunk, and lower- body interfaces now matter. The final outline contains golfer, club, and ground contact. Muscle work is internal, while gravity and environmental contacts are external. Each outline describes the same physical event, yet the words "source" and "transfer" refer to different terms.

This explains why two apparently contradictory papers can both balance. One may call power at a joint a transfer between rigid segments; another may treat the same joint and its spanning muscles as internal to a larger system. A useful comparison first translates both analyses to a common boundary and reference point. Without that translation, disagreement may be bookkeeping rather than mechanics.

2.2.5 A Boundary Audit in Plain Language

For any claimed transfer, write a one-line sentence: "The system is ***; energy crosses through*** ; positive points _____." Next list the stored forms. For a rigid club those might be translational and rotational kinetic energy plus gravitational potential. For a flexible club, add modal or strain energy. Then list external ports: hands, gravity, air, and ball contact. The balance should read like a bank statement in which the change in account value equals deposits minus withdrawals, with a small numerical residual.

Reference points deserve equal care. A force applied away from the club's center of mass creates both translation and rotation. Moving the reporting point changes the force–moment pair, although the physical wrench is the same. Power remains consistent only when force, moment, linear velocity, and angular velocity are transformed together. Mixing a force from one point with a moment from another is the mechanical equivalent of combining dollars and euros without an exchange rate.

2.2.6 Why the Choice Matters for the Two-Hands Question

When the hands exert equal and opposite forces, their net force can be nearly zero while their couple is not. For the club-only system, that couple is an external moment and can do rotational work. For a hands-plus-club system, the same contact forces are internal, and the energetic question moves to the wrist/arm boundary. Neither description makes the couple disappear. It changes where the accounting line is drawn.

The distinction is essential when comparing an arm-dominant allocation with a wrist-dominant allocation. If both produce the same net club wrench, club-level motion cannot identify which internal route was used. Expanding the system may make the hidden route explicit, but it also demands more measurements and more anatomical assumptions. A good model chooses the smallest boundary capable of answering its declared question and refuses to imply more.

2.3 Where the Picture Breaks

The household-budget analogy suggests that every transfer has a unique payer and recipient. Multibody mechanics is less tidy. Coordinate choices can move terms between what is called inertial coupling and what is called a generalized force. Segmental energy methods can partition joint power in more than one way, especially at multi-degree-of-freedom joints. Soft tissues span several joints, and biarticular muscles can move energy between segments without fitting a single-joint wallet.

Rigid-body boundaries also cut through biological structures. A modeled "wrist joint" may stand in for carpal motion, forearm rotation, grip deformation, and small translations. A "shoulder torque" may aggregate glenohumeral action, scapulothoracic motion, and trunk coupling. The boundary remains useful if its meaning is explicit, but it should not be mistaken for a direct anatomical measurement.

Finally, work and energy say nothing by themselves about comfort, accuracy, injury risk, neural control effort, or repeatability. Two allocations can have the same club energy and very different internal loads. The later chapters add those objectives rather than letting one ledger pretend to answer them all.

Go Deeper: The scientific monograph's whole-system and interface-power treatment shows the balance for rigid and flexible cases. Its two-hand wrench chapter defines reference points and action-reaction explicitly. For the broader biomechanics convention, see Winter's treatment of segment and joint energetics (Winter 2009).

Chapter 3

The State Is the Complete Snapshot

3.1 A Concrete Picture

Two photographs can look identical and conceal opposite futures. In the first, a playground swing has reached its highest point and is about to fall. In the second, it is moving upward through the same angle and is about to slow. The seat, chains, and child occupy nearly the same places in both photographs. Yet one moment leads downward and the other upward because velocity is missing from the picture.

Now imagine two bows held in the same visible shape. One bowstring has been drawn and carries elastic tension; the other is a decorative loose cord. Their geometry can match while their stored energy differs. Or imagine two muscles at the same joint angle: one is activated and stiff, the other relaxed. Position alone again fails to tell us what happens after a disturbance.

A useful dynamics model gathers everything it needs to continue the motion into a **state**. For a rigid double pendulum, the state may consist of two angles and two angular velocities. For a flexible club, it also includes shaft deflection and deflection rate. For a delayed actuator, it may include internal command or activation state. For a contact model with preload, it includes the current interface deformation. The complete snapshot is more than a pose.

3.2 How the Mechanism Works

The formal state is commonly written $\mathbf{x}$. For a two-link planar model, one convenient choice is

$$\mathbf{x} = [q_1, q_2, \dot{q}_1, \dot{q}_2]^\intercal,$$

where q_1 and q_2 describe configuration and the dotted variables describe angular velocity. The equations of motion define a rule

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, t),$$

which says: given the present state, the present control $\mathbf{u}$, and any explicit time dependence, compute

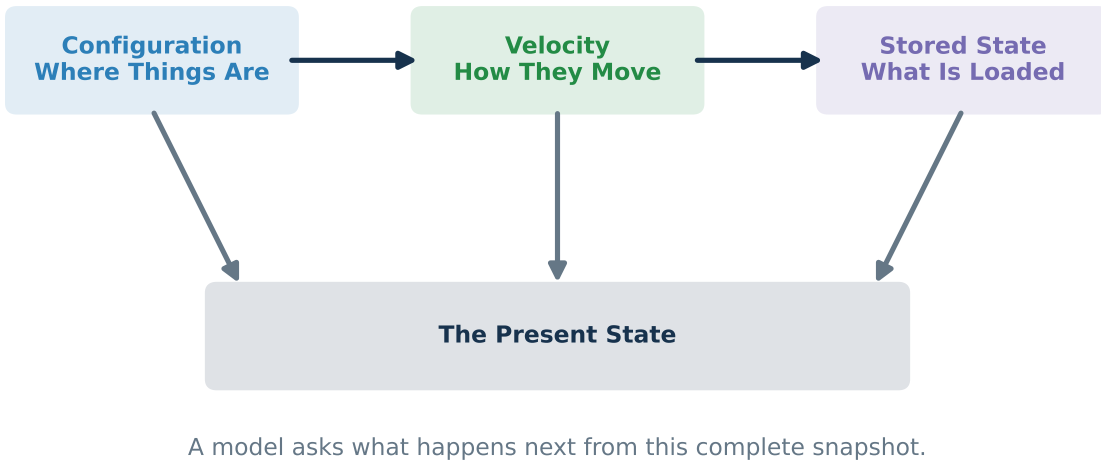

Figure 3.1: Configuration, Velocity, and Stored Internal Variables Join to Form the State Needed to Predict What Happens Next.

the rate at which the state changes. A numerical integrator advances that rule through many small steps.

The word *state* has a strict purpose. If two simulations begin with the same state and receive the same future inputs under the same deterministic model, they should produce the same future. If an omitted variable allows two different futures, the declared state was incomplete for that model. This is why a shaft angle without its angular rate is insufficient, and why an interface position without its stored preload may be insufficient.

3.2.1 Configuration Is More Than a Photograph

Configuration describes where the modeled bodies are within their allowed geometry. In a planar double pendulum, two angles may be enough. In a spatial golfer model, configuration can include pelvis position and orientation, spinal and shoulder rotations, elbow and wrist coordinates, club pose, and perhaps shaft modes. Rotations require care because three-dimensional orientation is not just three independent slider positions. Different angle sequences can describe the same physical orientation, and some parameterizations contain singularities.

Configuration also determines mechanical advantage. The mass matrix, moment arms, constraint Jacobian, gravity direction, and mapping from joint torques to club acceleration all change with posture. The same torque vector applied at two configurations can produce different accelerations. State-dependent geometry is therefore not scenery around the mechanism; it is part of the mechanism.

3.2.2 Velocity Gives the Snapshot a Direction

Velocity tells how configuration is changing. It carries both magnitude and direction. Two systems at the same pose with opposite velocities have different momentum, different convective forces, different power at their interfaces, and different short-term futures. In linked rotational systems, products of angular velocities create Coriolis and centrifugal contributions. Those terms can be large late in a swing even if the active torque is small.

Velocity is also needed to evaluate power. A 100-newton force at a stationary point delivers zero instantaneous translational power. The same force at a point moving 5 metres per second in its direction delivers 500 watts. Reverse the velocity and the power is −500 watts. Force without velocity cannot tell the energy story.

3.2.3 Stored Variables Carry History Into the Present

Rigid-body coordinates and velocities are not always enough. A flexible shaft has modal displacement and velocity. A series-elastic actuator may have tendon stretch. A first-order activation model has a state that lags its neural command. A contact dead zone has a current deflection that decides whether the interface is engaged. These variables are **history summarized**: they contain just enough of the past to determine how the model continues.

This matters directly to the proposed preload mechanism. If one simulation begins the downswing with an arm and wrist transmission channel already loaded, while another begins with both channels relaxed, those are not the same state. Comparing their futures cannot isolate the effect of a new command alone. The technical study therefore added a finite preparation interval beginning from zero internal deflection. It let both strategies build their own internal state and then carried that state continuously across the transition. No hidden reset was allowed.

3.2.4 State and Input Must Not Be Confused

An input is something the model treats as externally prescribed for the future: a joint torque command, a desired actuator level, or a measured base trajectory. A state is something the model evolves. The distinction can change between model tiers. A prescribed hand path is an input to an inverse-dynamics model; the same hand path becomes an outcome in a forward model where joint torques are inputs. A prescribed pelvis motion can hide the reaction of the club on the body; a moving-base forward model lets that feedback alter the state.

This difference explains why two models can answer different questions even if they draw the same stick figure. An inverse model asks what forces are compatible with measured motion. A forward model asks what motion results from forces and an initial state. Neither is automatically superior. They have different estimands and different opportunities for circular reasoning.

3.2.5 A Matched-State Intervention

Suppose we want to ask what the current shoulder torque contributes to club acceleration. One useful experiment freezes the state at an instant and solves the acceleration twice: once with the recorded torque and once with that torque set to zero. Because configuration and velocity match exactly, the acceleration difference is an instantaneous control contribution under the model. This is a pointwise counterfactual.

If we then integrate the zero-torque branch forward, its state begins to diverge. After a few milliseconds, force and power differences include both the missing torque and the changed trajectory it caused. That forward counterfactual answers a persistence question rather than a pure instantaneous attribution question. Both are legitimate if their state contracts are stated.

3.2.6 Phase Labels Are Functions of State

Golf language uses phases such as transition, early downswing, delivery, and impact. A clock time alone may not align these phases across people or model runs. One swing reaches a club angle sooner; another has a different peak hand speed. Event-based analysis defines phase landmarks from state: a zero crossing, an angle, a velocity peak, or the first valid impact condition. This can make comparisons more meaningful, but event definitions must be frozen before looking at outcomes or they become another adjustable explanation.

Model Result: The matched-state and continuous-preparation studies preserve the complete declared state at their intervention boundaries. Their differences therefore have a narrower interpretation than comparisons between unrelated full swings.

Human Evidence: Motion capture estimates configuration and velocity, but soft-tissue artifact, differentiation noise, and unmeasured internal states limit state reconstruction. Shaft strain, grip pressure, EMG, and ultrasound can add pieces; none alone produces a complete biological state (McPhee 2022).

Hypothesis: A golfer may use backswing loading and early-downswing motion to arrive at a state from which interaction dynamics assist late club acceleration. That is a state-arrangement hypothesis, not merely a claim about waiting a fixed number of milliseconds.

Practical Interpretation: A pose is the cover of the book, not the page you are on. To understand what comes next, ask how everything is moving and what internal elements are already loaded.

3.2.7 Three Identical Pictures With Different Futures

Picture three model snapshots in which the arm and club have exactly the same angles. In the first, both links are almost still. In the second, the arm is rotating rapidly while the club lags. In the third, the arm is slowing while the club is already releasing. A photograph cannot distinguish them, but the mass- matrix and velocity-dependent terms immediately can. If all applied torques are removed for the next few milliseconds, the three systems follow different paths because their velocities differ.

Now give the club a flexible shaft. The grip and clubhead may occupy the same visible positions in two trials while the shaft bends in opposite modal phases. One is poised to return energy in the swing direction; the other may initially absorb it. Add muscle activation and short-range stiffness, and two outwardly similar states can also resist perturbation differently. "Same position" is therefore far weaker than "same state."

3.2.8 Why Matched-State Comparisons Are Powerful

Suppose we want to know what a wrist torque contributes at time t^*. Comparing two ordinary swings that happen to pass through similar poses is inconclusive: their velocities, shaft states, and previous controls may differ. A matched- state intervention takes one simulated state at t^*, copies it, changes only the selected input, and integrates both copies. The initial difference is then known exactly. The trajectories diverge because of the intervention and its later interactions with the dynamics.

This is the forward counterpart to a pointwise decomposition. The pointwise question asks what acceleration contribution exists at the copied instant. The forward question asks whether that local contribution matters after the system evolves. A component can be large and fleeting, or small and persistent. Both tests are needed because a late golf downswing provides little time for a local effect to accumulate.

3.2.9 State-Based Timing Versus Clock-Based Timing

A command issued 120 milliseconds before impact is not necessarily equivalent across golfers or clubs. One system may already have a small arm–club angle and high relative speed; another may still be carried with substantial lag. A state- based rule might instead refer to relative angle, angular velocity, shaft mode, or the sign of an interaction term. Such a rule is still a hypothesis, but it can travel across changes in overall tempo more sensibly than a fixed clock delay.

The backswing-preload proposal belongs here. Its plausible benefit is not that one direction of effort is inherently superior. It is that continuous loading might deliver a transition state with less force-development delay than a full role reversal. To test that claim, the state must include the modeled transmission or activation variable. If the model omits it, the claimed benefit cannot appear except by assumption hidden elsewhere.

3.2.10 What Must Be Measured

Motion capture supplies positions and derived velocities, but filtering choices matter greatly near transition. Shaft strain or high-speed shape reconstruction adds deformation state. Bilateral grip sensors add contact wrench. EMG provides activation timing but not muscle force directly. A defensible experimental state is therefore an estimated distribution with uncertainty, not a perfectly known row of numbers. Counterfactual conclusions should be repeated across that distribution to learn whether they survive plausible reconstruction error.

3.3 Where the Picture Breaks

The "complete snapshot" phrase can suggest that a finite model state contains all physical reality. It does not. A model state is complete only relative to the equations chosen. A two-link rigid model omits shaft vibrations, soft tissue, muscle activation, grip deformation, aerodynamics, and neural feedback. Its four-number state can perfectly determine the future of that reduced model while remaining a sparse description of a human swing.

Biological systems may also contain memory that simple first-order states do not capture: muscle history dependence, reflex delays, thixotropy, fatigue, and learning. Adding every possible state would make estimation impossible. The scientific task is to add a state when it changes the question or survives a falsification test, not because complexity sounds realistic.

Finally, measurements never reveal state without uncertainty. Velocities are often derived from noisy positions. Internal force and activation states may be only indirectly observed. A responsible model carries those uncertainties into its conclusions instead of treating one reconstructed trajectory as exact.

Go Deeper: See the monograph's matched-state persistence chapter, the continuous preparation experiment, and the interactive workbench for guided state-versus-clock interventions.

Chapter 4

Geometry Is Part of the Machine

4.1 A Concrete Picture

Open a heavy door by pushing near the hinge. Then push with the same effort near the handle. Your muscles may feel the same force, but the door does not. The distance from the hinge changes the turning effect. Now push the handle directly toward the hinge. The force may be large and the handle may hurt your palm, yet the door scarcely rotates because the force line passes close to the axis.

Nothing about the door's motor changed; there is no motor. Geometry changed how force mapped into rotation. A golf swing is full of moving versions of that door. The hand path, club angle, arm orientation, grip separation, joint axes, and shaft bend change moment arms continuously. The same muscle or joint torque can assist one coordinate, oppose another, or mostly create internal load as the configuration evolves.

Geometry is therefore not a picture drawn after the dynamics calculation. It is inside the calculation.

4.2 How the Mechanism Works

For a force $\mathbf{F}$ applied at position $\mathbf{r}$ relative to a reference point, the moment is

$$\tau = \mathbf{r} \times \mathbf{F}.$$

The cross product contains both magnitude and direction. Its magnitude is $rF \sin\theta$, where θ is the angle between the position vector and the force. A force along the radius has $\sin\theta = 0$ and creates no moment about that point. A perpendicular force has the largest moment for fixed r and F. This simple relation powers much of the later two-hand argument.

In a planar model, the cross product becomes a signed scalar. If the club points along unit vector $\hat{\mathbf{e}}_r$ and its swing-normal direction is $\hat{\mathbf{e}}_t$, a hand force can be decomposed as

$$\mathbf{F} = F_r \hat{\mathbf{e}}_r + F_t \hat{\mathbf{e}}_t.$$

F_r acts along the club. It contributes to axial tension or compression and may strongly constrain the

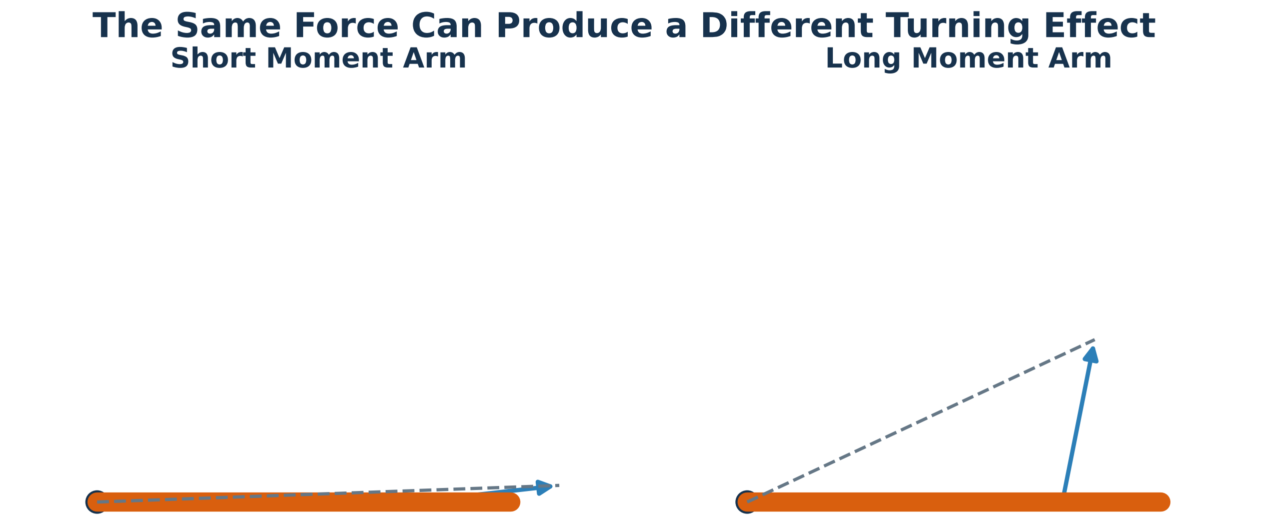

Figure 4.1: Force Direction, Moment Arm, and the Chosen Reference Point Determine the Turning Effect on the Club.

path, but about a point on the same club line it has little moment. F_t acts across the club and creates a turning effect with a moment arm set by distance along the shaft. As the club rotates, the global horizontal and vertical components change even if F_r and F_t do not. That is why frame declarations matter.

4.2.1 Coordinate Frames Are Different Viewpoints

Imagine giving directions in a city. "North" belongs to a fixed map frame. "Forward" belongs to the car and changes as the car turns. Neither is wrong, but mixing them mid-sentence creates confusion. Swing mechanics commonly uses a laboratory frame fixed to the room, a body or segment frame rotating with each link, a club frame aligned with the shaft, and a swing-plane frame chosen from the motion.

A physical force is the same interaction in all frames, but its numerical components change through rotation. A positive global vertical force may have a negative club-tangential component at one orientation and a positive one later. A report that calls a force "negative" without naming axis and frame leaves the reader unable to tell whether the physics reversed or only the coordinate description changed.

Moments add another reference. A wrench expressed about the lead hand differs from the wrench about the club center because moving the reference point adds the moment of the resultant force. When two hands apply nearly opposed forces, the resultant may be small while the couple remains large and nearly independent of reference point. When the resultant is not small, the reference transformation is essential.

4.2.2 The Jacobian Is a Geometry Translator

In multibody mechanics, a Jacobian maps generalized velocities to the velocity of a point or body. If $\dot{\mathbf{q}}$ contains joint rates and $\mathbf{v}_H$ is hand velocity, then

$$\mathbf{v}_H = \mathbf{J}_H(\mathbf{q})\dot{\mathbf{q}}.$$

The transpose maps a hand force back to generalized joint forces:

$$\tau_q = \mathbf{J}_H(\mathbf{q})^\mathsf{T}\mathbf{F}_H.$$

The matrix depends on configuration. It is the moving collection of moment arms that translates between joint-space and Cartesian-space descriptions. Near a singular configuration, a small Cartesian task may require large joint motion or force, and some internal directions may become poorly identifiable. A model that ignores this geometry can mistake a difficult posture for weak actuation or an easy posture for better timing.

4.2.3 Inertia Also Changes With Geometry

The mass matrix $\mathbf{M}(\mathbf{q})$ describes how generalized acceleration relates to generalized force at a configuration. Its off-diagonal terms are the mathematical signature of inertial coupling. In a double pendulum, accelerating the arm affects the club coordinate even with zero direct wrist torque. The strength and sign of that effect change with relative angle.

This is the deeper reason the linked system is not two separate motors. One joint's torque can cause acceleration at another coordinate because the bodies share mass and constraints. At one relative angle, proximal acceleration may help uncock the club. At another, the same command may resist it. Velocity terms change too, so geometry and momentum work together rather than taking separate turns.

4.2.4 Grip Separation Creates a Lever for a Couple

Two hands introduce a short but important distance. Suppose the lead and trail hands apply equal and opposite swing-normal forces $+F$ and $-F$ separated by distance d along the grip. Their net force is zero, but their net couple has magnitude

$$M = Fd.$$

Double the separation and the same forces create twice the moment. Reverse which hand pushes in which direction and the moment changes sign. Make the two contact points coincide and the force-generated couple must vanish. These are powerful geometric negative controls: if a simulated couple survives coincident contacts or reversed moment arms without the expected change, something is wrong in the implementation or interpretation.

The real hands do more than apply two ideal point forces. Pressure spreads over the grip, wrists can apply direct free moments, and the shaft deforms. The equivalent wrench still lets us compare the net effect at a declared reference. It separates what is observable at the club level from how the biological system allocated forces internally.

4.2.5 Geometry Can Change the Best Allocation

The arm-wrist allocation study holds an 8 N m club-moment task fixed while varying club angle and the fraction assigned to direct wrist moment versus a force-generated two-hand couple. Every allocation closes the same task within numerical precision, yet RMS hand force and generalized joint-torque norm vary widely. The allocation minimizing one metric shifts with club angle and differs from the allocation minimizing another metric.

That result is modest but important. There is no geometry-free answer to "use the arms or use the wrists." Even within a reduced model, the internal cost depends on orientation and on which cost is valued: hand force, joint torque, work, stability, accuracy, or discomfort. Human anatomy adds more moving axes and constraints, not fewer.

Model Result: Same-state force and torque mappings change with club angle, hand separation, and moment-arm direction. Reversing or collapsing those geometries provides exact negative controls in the two-hand studies.

Human Evidence: Golf kinematics shows meaningful variation in hand path, shaft plane, wrist angles, and body orientation across golfers. Those measurements establish geometric diversity but do not by themselves identify the forces or muscle allocations producing it (McPhee 2022).

Hypothesis: Late-downswing negative coupling may arise in part because the two-hand force geometry projects interaction forces into a negative club moment near the impact zone. That mechanism should change predictably when moment arms or contact geometry are reversed.

Practical Interpretation: Direction and leverage can matter as much as effort. Before asking how hard a force is, ask where it acts, which way it points, and about which point its turning effect is being measured.

4.2.6 Read a Force Vector in Three Passes

First read its line of action. A force parallel to the club shaft mainly loads the shaft and grip axially in a simple planar picture; a perpendicular component has more immediate turning leverage about the club's center of mass or a chosen joint. Second read the moment arm: the perpendicular distance from the reference point to the force line. Third read the motion: power depends on whether the contact point moves along or against the force. Magnitude alone answers none of these questions.

Consider a 100-newton force acting with a perpendicular moment arm of 0.04 metres. Its turning effect is 4 newton-metres. If posture changes the moment arm to 0.08 metres without changing the force, the moment doubles. If the force line passes through the reference point, the moment becomes zero. If the hand order or force direction reverses, the moment changes sign. These simple controls are why geometry reversal is a strong falsifier of a proposed two-hand mechanism.

4.2.7 Geometry Changes During the Swing

The relevant moment arm is not fixed on a diagram. Arm–club angle, hand separation, wrist orientation, swing-plane normal, and shaft deflection all evolve. A force that is mostly radial early can acquire a tangential projection later. A pair of forces that makes a negative swing-normal couple in one state may make a smaller couple, no couple, or the opposite couple after the contact frame rotates. Consequently, a statement such as “the trail hand pushes” is mechanically incomplete until direction, point, coordinate frame, and phase are specified.

The mass matrix carries another form of geometry. Its off-diagonal terms encode how acceleration of one coordinate affects generalized momentum associated with another. Those terms vary with joint angle. Velocity-dependent terms contain products of speeds and trigonometric factors, so both state and geometry decide their sign. Interaction is not a mysterious extra force inserted by a modeler; it is what the equations require when linked bodies move in changing geometry.

4.2.8 A Two-Hand Thought Experiment

Place two contact points on a rigid handle, separated by distance d. Apply equal opposite forces of magnitude F perpendicular to their separation. Net force is zero, yet the couple magnitude is approximately Fd. Halve the separation and the couple halves. Rotate the forces until they become parallel to the separation vector and the couple collapses. Swap the force directions and its sign reverses. These are not coaching claims. They are geometric predictions any correct reconstruction must satisfy.

Now add a common force to both hands. That common mode translates the club and can also create a moment depending on its line of action. The measured club wrench combines common and differential modes. Without individual hand sensing, many allocations reproduce it. Geometry tells us what each proposed allocation would do, but net motion cannot select the biological one.

4.2.9 Geometry as Strategy Constraint, Not Prescription

For speed, one might seek states where proximal motion and constraint forces project favorably onto club motion. For control, one might prefer states with less sensitivity to small timing or force errors. For comfort, the preferred geometry may avoid extreme joint positions. These objectives can conflict. A scientific strategy study therefore maps a surface—speed, face/path, internal load, and robustness—rather than announcing one aesthetically pleasing pose.

4.3 Where the Picture Breaks

The door analogy has one fixed hinge and one obvious plane. Human joints have multiple axes that can move relative to one another. The scapula glides and rotates on the thorax; the shoulder center is not a nailed hinge; forearm pronation changes wrist-axis orientation; the grip is distributed and compliant. A planar moment arm can reveal a mechanism without reproducing this anatomy.

Jacobian and mass-matrix descriptions also depend on coordinates. Their entries can change dramatically under a reparameterization even when physical motion is unchanged. Conclusions should therefore be tied to frame-invariant observables such as reconstructed wrenches, power, work, and trajectory, not to the apparent size of one matrix term alone.

Finally, geometry does not dictate a unique strategy. It constrains and weights possibilities. A posture that improves mechanical advantage might reduce range of motion, increase tissue load, or worsen face control. Selecting a human technique requires objectives and evidence beyond one moment arm.

Go Deeper: Read the technical two-hand wrench derivation, the matched allocation surface, and the three-dimensional frame audit.

Chapter 5

Speed, Acceleration, Energy, and Power

5.1 A Concrete Picture

A car dashboard separates quantities that casual speech blends together. The speedometer says how fast the car is moving now. Pressing the accelerator changes speed, but the rate of change is acceleration, not speed itself. The fuel gauge tracks stored chemical energy, not motion. The engine's power rating describes how rapidly it can do work under stated conditions. A heavy truck and a bicycle can show the same speed while carrying very different kinetic energy.

Swing discussions often collapse these distinctions into "speed." A pelvis can have a high angular speed but modest kinetic energy because its effective rotational inertia about a chosen axis differs from the club's. A club can have positive acceleration while receiving negative power if changing geometry redistributes kinetic energy among coordinates. A large instantaneous power peak can deliver little work if it lasts only a moment. Before following transfer, we need separate meters.

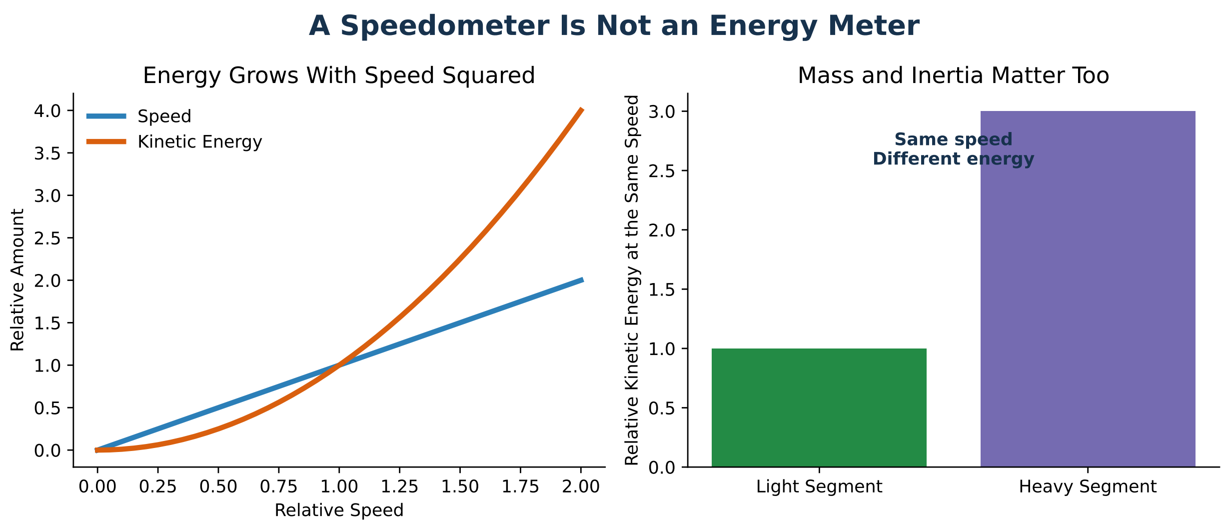

Figure 5.1: Speed and Kinetic Energy Rise Differently, and Bodies at the Same Speed Can Carry Different Energy.

5.2 How the Mechanism Works

Velocity is motion with direction. Translational velocity is measured in metres per second; angular velocity in radians per second. The clubhead velocity is not simply the wrist angular velocity multiplied by club length because the wrist itself is moving and the arm-club geometry changes. In a linked chain, endpoint velocity is a geometric combination of all relevant joint rates.

Acceleration is the rate of change of velocity. A body moving around a circle at constant speed is accelerating because direction changes. This distinction is critical near impact: large radial acceleration and tension can exist without a corresponding increase in speed. The force responsible for bending the path can be enormous while its instantaneous work is small.

Kinetic energy for a translating point mass is

$$K = \frac{1}{2}mv^2.$$

For rotation about a fixed axis it is $K = \frac{1}{2}I\omega^2$. The square means a 10% speed increase produces about a 21% kinetic-energy increase if mass and inertia remain fixed. In a multibody system, kinetic energy is more generally

$$K = \frac{1}{2}\dot{\mathbf{q}}^{\mathsf{T}}\mathbf{M}(\mathbf{q})\dot{\mathbf{q}},$$

so cross-terms capture shared motion and configuration-dependent inertia. Assigning one independent energy number to each angle can miss this coupling.

Power is the time derivative of work or energy transfer. A 1,000 watt input maintained for 0.05 seconds delivers 50 joules. A 5,000 watt spike lasting 0.002 seconds delivers only 10 joules. Peak power attracts attention, but integrated work answers how much energy crossed the boundary during a phase.

Torque power is $P = \tau\omega$ in one dimension. Four sign combinations are possible. Positive torque with positive angular velocity supplies positive power. Negative torque with positive velocity removes energy. But negative torque with negative velocity again supplies positive power. “Negative torque” is a direction statement, not an energy verdict.

5.2.1 Segment Speed Peaks Are Landmarks, Not Packages

If pelvis, trunk, arm, and club angular-speed peaks occur in order, the sequence describes coordination. It can indicate that distal segments continue to accelerate after proximal segments begin slowing. Yet the energy route remains underdetermined. A proximal segment may slow because it does negative work, because interaction forces remove energy, because its effective inertia changes, or because energy is redistributed within the linked kinetic-energy expression.

A clean analysis overlays segment energies, joint and interface powers, and external work. When the club gains energy, we ask which powers integrate to that gain. When a segment slows, we do not automatically label its lost kinetic energy as transferred until the boundary ledger supports that statement.

5.2.2 A Worked Comparison

Consider two idealized clubheads of equal mass. Club A reaches 40 m/s and Club B reaches 44 m/s. The speed difference is 10%. Their translational kinetic energies differ by $(44/40)^2 = 1.21$, or 21%. That extra energy must be supplied, released from storage, or retained by reducing losses. A model that predicts a small speed increase should therefore pass a demanding work-energy check.

Now compare a 0.20 kg clubhead at 50 m/s with a 70 kg golfer center of mass moving at 1 m/s. The simplified translational energies are 250 J and 35 J, respectively. The slower object can have less energy despite far more mass, while body segments also carry substantial rotational energy. Speed alone cannot rank their energetic importance.

5.2.3 Acceleration Decomposition Is Not Energy Decomposition

The equations can decompose acceleration into gravity, velocity-dependent, control, and constraint contributions at a fixed state. Those contributions add linearly at that instant. Their powers and accumulated works require velocities and time histories. A term that contributes positive club angular acceleration for one instant need not deliver positive net work across the impact window.

This is why the technical paper reports impulse, work, and power alongside acceleration. Impulse measures accumulated force over time; work weights force by displacement; power shows timing. Together they prevent a single attractive peak from standing in for the mechanism.

Model Result: Late wrist drive improves club speed in the declared double- pendulum schedules, while an early opposing command can improve one lower-tail speed metric in an uncertainty ensemble. Neither result is interpretable from torque sign alone; the power and trajectory histories determine the outcome.

Human Evidence: Kinematic sequences and clubhead speed are measurable, but estimating energy and power requires segment inertias, joint definitions, forces, and filtering. Reported relationships vary across methods and populations (Tinmark et al. 2010; McPhee 2022).

Practical Interpretation: Use four separate questions: How fast is it? How is its velocity changing? How much mechanical energy does the chosen system have? How rapidly is energy entering or leaving through each boundary?

5.2.4 One Downswing, Four Different Graphs

A clubhead-speed graph tells us the magnitude of endpoint velocity, not which way the clubhead is moving or how that motion was produced. An acceleration graph can change sign while speed still rises because tangential and normal components play different roles. Even motion at constant speed requires inward acceleration when the path curves. Treating acceleration as merely "more speed per second" hides that geometry.

An energy graph weights speed by inertia. For a rigid segment rotating about a fixed point, rotational kinetic energy is one half of inertia times angular speed squared. A linked club does not rotate about one fixed hinge; its center translates while the body rotates. A complete calculation includes translational and rotational terms and avoids double counting. Because speed is squared, a small late speed increase can require a larger energy increase than the same numerical speed increase early.

A power graph answers a different question. Positive power means energy is entering the selected system through the named port at that instant. Negative power means it is leaving. Large alternating positive and negative peaks can integrate to modest net work. Conversely, moderate positive power sustained over a long interval can contribute substantial work. The area under a power curve matters alongside its peak.

5.2.5 Torque Sign Is Not Power Sign

Rotational power is the dot product of torque and angular velocity. In a planar coordinate it reduces to $P = \tau\dot{q}$. If both torque and angular velocity are positive, power is positive. If torque is negative while the coordinate still rotates positively, power is negative. If the coordinate reverses too, a negative torque can have positive power. A label such as "negative wrist torque" is incomplete until the coordinate and angular velocity are named.

The same caution applies to a two-hand couple. A negative swing-normal couple can remove rotational energy from the club locally while other hand-force components supply positive translational power, the shaft returns energy, and proximal coupling changes future geometry. The final speed outcome belongs to the integrated coupled trajectory, not the sign of one channel.

5.2.6 A Worked Comparison

Imagine two late-downswing intervals lasting 0.05 seconds. In the first, hand power into the club averages 800 watts, so idealized work is 40 joules. In the second, power peaks at 1,500 watts but changes sign and averages only 300 watts, producing 15 joules. The larger peak did less net work. Now add a shaft returning 8 joules during the second interval and an aerodynamic loss of 2 joules in each. The full club-energy changes become roughly 38 and 21 joules. These illustrative numbers show why relevant ports must be integrated consistently.

Suppose the second strategy nonetheless ends with similar clubhead speed. It may have started with more stored kinetic or strain energy, changed club orientation to alter effective inertia, or traded another output such as face control. Equality of one terminal metric cannot identify the history. Time series and ledgers distinguish histories that a launch-monitor number collapses.

5.2.7 Why Impulse Belongs Beside Work

Impulse integrates force over time and changes linear momentum. Angular impulse integrates moment and changes angular momentum. Work integrates force along motion and changes energy. A force nearly perpendicular to point velocity may deliver substantial impulse with little work, redirecting motion more than speeding it. A constraint force can therefore be dynamically important even when its direct power is zero.

This distinction is central to transfer. The body can arrange and redirect the club's momentum through linked geometry while muscular work enters elsewhere. Calling every beneficial interaction an "energy source" is wrong; calling a zero-work force irrelevant is also wrong. Reporting force, impulse, power, work, and acceleration contribution keeps both errors visible.

5.2.8 A Reader's Diagnostic Checklist

Inspect units. Metres per second is speed; joules is energy; watts is power; newton-seconds is impulse. Check whether the quantity is signed or a magnitude, whether it is at a joint or endpoint, and whether it is normalized. Then look for the interval definition and numerical integral. If a conclusion about total work rests only on a power peak, or a conclusion about endpoint speed rests only on joint angular velocity, the argument is incomplete.

5.2.9 Reader Lab: Diagnose a Misleading Result

Suppose a report says one strategy is better because its clubhead-speed curve has a steeper late slope and its wrist-power peak is larger. Ask for the actual endpoint velocity vector, not only magnitude, and inspect whether both trials start the comparison window with equal energy. Integrate hand and wrist power, include shaft and aerodynamic terms, and compare residual closure. The larger peak might be shorter, partly canceled, or accompanied by a different initial state.

Next compare shot-relevant outcomes. Did face orientation, path, impact location, or variability change? Did the faster curve require greater peak hand force or joint load? A mechanical speed result can remain correct while the broader claim "better strategy" fails.

Finally, branch both simulations from an identical state and exchange only the control schedule. If the ranking persists under timestep refinement and parameter uncertainty, it is a robust model result. If it disappears, the original difference belonged to preparation or parameter choice. This simple lab shows why the four gauges—speed, acceleration, energy, and power—must be read together.

5.2.10 Language That Preserves the Distinction

Use "the clubhead speed increased," "the club mechanical energy changed," "the hands supplied positive power," and "the interface performed positive work" as separate sentences. State the system and interval. Avoid "power was stored," because power is a rate, and avoid "speed was transferred," because speed is not a conserved substance. Clear language is a scientific control: it makes a wrong inference harder to hide inside a familiar phrase.

5.3 Where the Picture Breaks

The dashboard analogy suggests independent gauges. In a linked body, the gauges are coupled. Effective inertia changes with configuration, endpoint velocity combines several coordinates, and segmental energy partitions can depend on convention. Biological energy also includes metabolic and elastic processes not captured by a rigid mechanical ledger.

Clubhead kinetic energy immediately before impact is not identical to useful shot outcome. Impact location, face orientation, path, dynamic loft, shaft state, and ball properties matter. A strategy that raises speed while worsening face or contact variability may lose on the actual task.

Go Deeper: See the monograph's mechanics and operational energy definitions, the phase-resolved results, and the foundational segment energetics literature (Robertson and Winter 1980; Winter 2009).

Chapter 6

Constraints Push Back

6.1 A Concrete Picture

Swing a key on a cord. Your hand does not continuously command the key's inward acceleration. The cord supplies tension because it prevents the key from flying away. Cut the cord and the key leaves along its instantaneous tangent. The constraint is not a passive absence of freedom; it is an interaction that pushes or pulls as needed to keep the allowed geometry.

Human joints, two hands on one grip, and feet on the ground all impose versions of this idea. A wrist reaction can be large even when commanded wrist torque is zero. Two arms can develop internal forces that cancel in the club's net force. The ground can exert a reaction determined by the whole body's state and constraints. "No motor command here" does not mean "no force here."

6.2 How the Mechanism Works

A holonomic constraint can be written $\phi(\mathbf{q}) = 0$. It says which configurations are allowed. Differentiating gives the velocity condition $\mathbf{J}\dot{\mathbf{q}} = 0$, where $\mathbf{J}$ is the constraint Jacobian. Differentiating again produces an acceleration condition involving $\mathbf{J}\ddot{\mathbf{q}}$ and a velocity term. The constraint force must be whatever value makes the equations of motion and this allowed acceleration agree.

One common system is

$$\begin{bmatrix} \mathbf{M} & -\mathbf{J}^{\mathsf{T}} \\ \mathbf{J} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q} - \mathbf{h} \\ -\dot{\mathbf{J}}\dot{\mathbf{q}} \end{bmatrix}.$$

λ contains constraint reactions. They are solved together with acceleration, not pasted on afterward. Change configuration, velocity, or control and the required reactions change.

6.2.1 Zero Torque Does Not Mean Zero Reaction

Set one selected control torque to zero at a fixed moving state. Gravity remains. Momentum remains. Other actuators remain. Most importantly, constraints remain. The wrist joint must still keep arm and club connected, so it transmits force. The resulting **zero-torque counterfactual force** is a

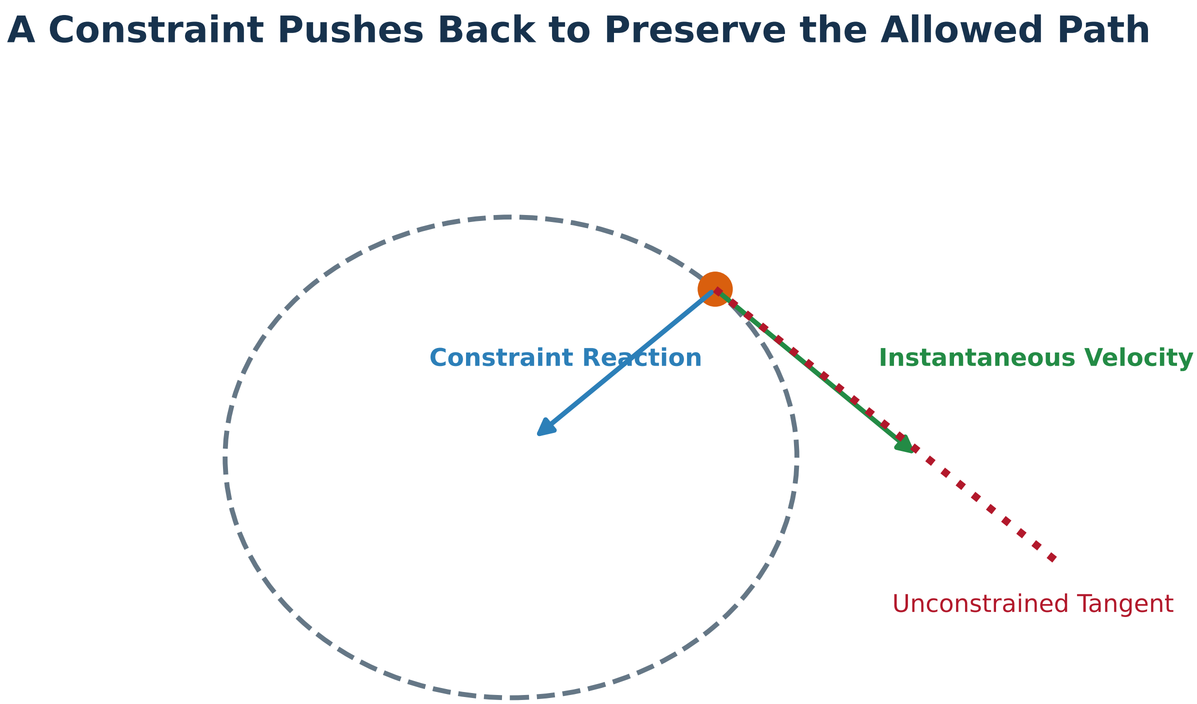

Figure 6.1: A Constraint Force Holds a Moving Mass to Its Allowed Path While an Unconstrained Counterfactual Leaves Tangentially.

model-defined answer to a narrow question: what reaction does this model require now if that selected torque is zero while the state is held fixed?

It is not "the passive force in the human wrist." It depends on which control was removed, which terms remain, and how the joint is modeled. Yet it reveals an important mechanism: interaction forces can arise from linked motion itself and can contribute substantially to the club's acceleration or hand-path work.

6.2.2 Ideal Constraints Can Carry Force Without Work

An ideal constraint force is often perpendicular to the allowed virtual motion, so it does no work on the complete constrained system. That does not make it unimportant. A rail pushes a train sideways around a curve. The normal force changes momentum direction while doing little work. At a subsystem boundary, the same interaction can have nonzero power on one part and opposite power on another.

Real biological contacts are not perfectly rigid or frictionless. Grip compliance, joint deformation, soft tissue, and shaft flex permit relative motion and energy storage. The ideal constraint is a reference case that separates geometric enforcement from compliance; it is not a claim that the interface has infinite stiffness.

6.2.3 Constraints Create Hidden Internal Modes

With two hands attached to one club, more contact-force components may exist than the club's net wrench can identify. Add equal and opposite forces to the two hands and the resultant can remain unchanged. That internal squeeze can alter joint loading and stability without appearing in club motion. Closed-chain inverse dynamics therefore cannot uniquely recover individual hand forces unless extra measurements or allocation rules are supplied.

The same issue appears at the feet. A net ground-reaction wrench may not uniquely identify left and right foot forces or pressure distributions. A force plate per foot, pressure insoles, and careful calibration reduce ambiguity, but model assumptions remain.

6.2.4 Reaction Forces Are Not Free Energy

Constraint reactions can accelerate one coordinate and oppose another. This can look like energy creation if only one segment is inspected. The whole-system work-energy balance restores perspective. The reaction redistributes momentum and energy under the constraint; active work, gravity, and stored energy still fund the ledger.

This is particularly important for the phrase "interaction forces increase transfer." They may increase club acceleration relative to a counterfactual by redirecting momentum or mediating power from proximal actuation. They do not manufacture energy. The scientific question is which state and geometry make their contribution favorable, for how long, and at what internal cost.

Model Result: In the double-pendulum decomposition, zero selected wrist torque leaves a substantial wrist reaction generated by gravity, velocity terms, proximal actuation, and the connection. Pointwise and forward tests quantify where its contribution persists.

Human Evidence: Joint reactions and individual hand forces are difficult to measure directly. Instrumented grips can constrain the two-hand problem, while motion and external-force data support inverse dynamics with stated uncertainty (Choi and Park 2020; Koike 2016).

Hypothesis: The late downswing may exploit constraint and inertial reactions so the club receives favorable acceleration even while a direct distal moment is small or opposing.

6.2.5 What a Constraint Actually Does

A constraint removes a motion that would otherwise be possible. A rigid pin keeps two points together while allowing rotation. An ideal no-slip contact keeps a foot point stationary relative to the ground. A bilateral grip model keeps specified hand and handle frames related. The equations must generate reaction forces large enough to satisfy those conditions while all masses, applied forces, and velocities evolve.

Those reactions are not corrections added after the motion. They are part of the simultaneous solution. Accelerations and constraint multipliers are solved together. Reaction at one joint therefore depends on applied torque elsewhere, gravity, configuration, velocity, inertia, and every other active constraint. This dependence is the mechanical foundation for interaction-force transfer.

6.2.6 A Wagon-and-Hinge Example

Imagine a light wagon connected by a hinge to a heavier powered wagon. Accelerate the powered wagon. The hinge pulls on the light wagon even though it has no motor. That force can increase its kinetic energy because the attachment point moves along the force direction. If the hinge instead forces a curved path, part of the reaction may redirect velocity with little direct work. The light wagon's motion is still caused in part by the connection.

Now brake the light wagon. The hinge reaction changes and affects the powered wagon. "Proximal causes distal" is therefore a useful phase description, not a one-way causal law. Linked dynamics is reciprocal. Distal speed often rises late because of evolving state and control, not because forces pass through a valve in only one direction.

6.2.7 Radial and Tangential Jobs

In a planar arm–club model, wrist reaction can be resolved along and across the club. The radial component helps enforce connection and supplies inward acceleration for curved motion. The tangential component more directly changes relative angular motion. Yet their labels rotate with the club, and a radial force at the wrist can contribute to club-center translation and system-level work. It is safer to calculate power and generalized acceleration than infer function from component names.

During early downswing, rapid arm motion can produce a reaction that carries the club while relative release remains limited. Later, changed geometry can project reactions more strongly into club rotation. A direct wrist command may supplement, oppose, or reshape that interaction. The model separates these terms at fixed state, then tests their persistence forward.

6.2.8 Pointwise Removal Versus Forward Removal

For a pointwise zero-torque counterfactual, copy one state, set the selected torque to zero, and resolve acceleration and reactions. This answers "what is this input contributing now under these equations?"

It does not say what a person would do after losing the input and does not allow state to change before comparison.

For a forward killswitch, both trajectories begin at the same state but one input is removed over an interval. They then occupy different states, so later differences include direct removal and nonlinear consequences of divergence. This is closer to causal intervention within the model but depends on the chosen continuation of all other controls. Agreement between local and forward stories is stronger than either alone.

6.2.9 When Reactions Become Ambiguous

If several contacts enforce the same net restriction, individual reactions may not be unique. Two hands can share a net force in infinitely many ways; two feet can share a ground wrench in many ways. A solver can choose a minimum-norm allocation, but that is a convention rather than a measurement. The null space should be reported or extra constitutive rules and sensors introduced.

Compliance changes the picture too. Real grip and shaft deformation turn an exact geometric constraint into a high-stiffness force law with state. That may smooth reactions, introduce delay, and store energy. A mechanism that survives both rigid and compliant representations is more credible than one existing only because an ideal constraint is infinitely stiff.

6.2.10 What Would Refute the Transfer Claim

The claim weakens if removing proximal drive at matched state produces no meaningful change in distal reaction or acceleration; reversing geometry does not reverse the predicted projection; work-energy closure fails; or the effect vanishes under finer integration and an independent solver. In human data, predicted reaction direction and timing must agree with synchronized hand wrenches and motion within uncertainty. Calling the reaction “passive” does not lower that evidentiary standard.

6.2.11 Reader Lab: Separate Motor and Connection

Run a two-link model from one copied state under four conditions: all inputs; proximal input only; distal input only; and neither input. At the copied instant, record direct generalized accelerations, constraint reactions, and endpoint acceleration. Over a short forward window, record power, work, and trajectory. The proximal-only branch should make visible how the connection moves the distal link without an explicit distal motor.

Now alter link angle while holding masses and instantaneous inputs. The reaction and its distal projection should change. Reverse a coordinate sign and transform all related quantities; physical endpoint motion and power should not change. Replace the ideal joint with a stiff compliant element and test convergence as stiffness rises. These operations distinguish a real coupling effect from one created by notation or constraint implementation.

The exercise also demonstrates cancellation. A direct distal input may oppose the proximal-generated reaction at one state. Total acceleration can be smaller than either component. That is why components are not percentages and why a negative contribution can still be part of a later-beneficial schedule.

6.2.12 The Everyday Meaning of Passive

In ordinary speech, passive means doing nothing. In mechanics, it often means a force follows from state, geometry, material law, or constraint rather than an explicit selected control. A spring force is passive even if it is large; a joint reaction is passive in that narrow channel even while motors elsewhere work hard. Every use in this book carries that technical meaning. Human intent and muscle effort require separate evidence.

6.2.13 One Sentence to Retain

A connection can strongly affect motion without being an independent energy source. Its force depends on the whole state and can transmit, redirect, store, or constrain the consequences of work performed elsewhere. That statement is both more accurate and more testable than saying the club accelerates "by itself."

6.3 Where the Picture Breaks

The cord-and-key picture has one exact constraint and no anatomy. A wrist has several degrees of freedom, ligaments, muscle stiffness, and distributed contact. The shoulder girdle moves. Grip friction can slip. A rigid planar hinge can test a coupling mechanism but cannot estimate tissue loading or injury risk.

Constraint-force decomposition can also depend on the mathematical partition. At redundant contacts, λ may be nonunique. Reporting one minimum-norm solution does not make it the biological allocation. The model must fail closed or show the null space.

Go Deeper: Study the exact wrist-reaction decomposition, the forward two-hand constraints, and the ground-reaction attribution.

Chapter 7

From One Pendulum to Two

7.1 A Concrete Picture

A single playground swing has one dominant rhythm. Add a loose second link at the seat and the motion changes character. The outer link can trail, fold, accelerate relative to the first, and feed reactions back into it. There is no moment when the first link acts alone and then politely hands control to the second. Both influence one another throughout.

The golf double pendulum replaces the inner link with an arm-like segment and the outer link with a club-like segment. It is deliberately spare. That is its strength: it is the smallest familiar model that contains changing geometry, inertial coupling, interaction force, relative-angle retention, and distal release.

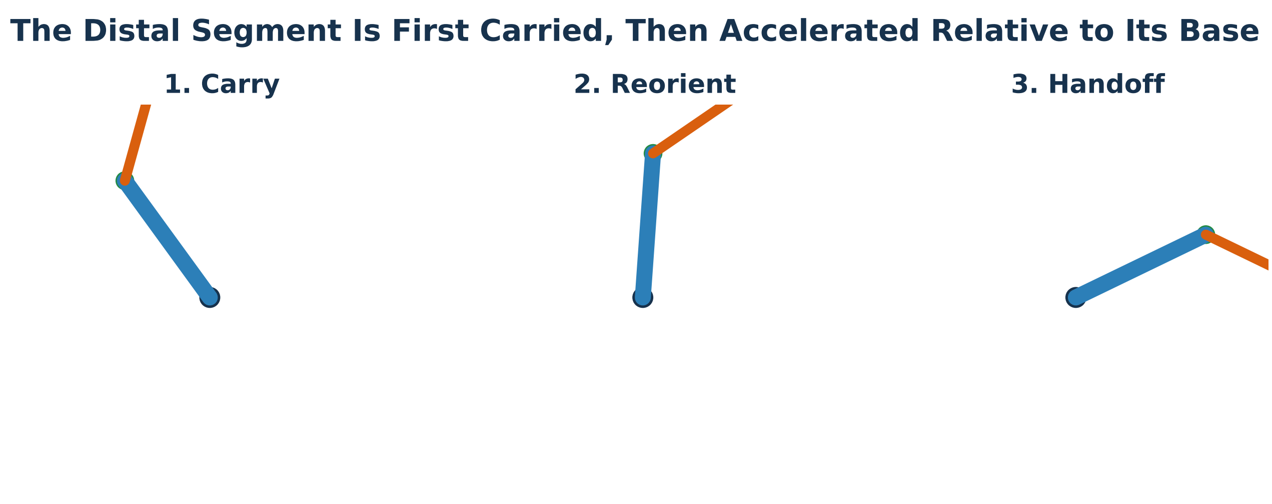

Figure 7.1: Three Snapshots Show the Club First Carried by the Arm, Then Reoriented, Then Accelerated Relative to the Moving Hand.

7.2 How the Mechanism Works

Let q_1 describe the arm angle and q_2 the club angle. The equations have the standard multibody form

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \tau.$$

The diagonal mass-matrix terms resemble each link's own inertia. The off-diagonal terms couple them. $\mathbf{h}$ collects gravity and velocity-dependent effects. Even with zero direct club torque, shoulder torque and the moving state can produce club angular acceleration because $\mathbf{M}^{-1}$ is generally not diagonal.

7.2.1 Carrying Is Not Wasted Motion

Early in many downswing patterns, the arm and club rotate more nearly together while the wrist angle remains relatively closed. The hand translates rapidly, and the club's center of mass gains motion even before large relative club rotation. Calling this only "holding lag" misses the transport: the distal body is being carried by a moving base.

As geometry changes, coupling can favor rapid relative rotation. The club's angular speed about the hand then rises and adds to hand velocity at the clubhead. The visible release is not energy appearing at the wrist. It is the outcome of earlier work, current geometry, interaction forces, direct torque, gravity, and possibly shaft storage.

7.2.2 Relative Angle Changes the Coupling

When links are nearly aligned, their combined effective inertia and moment arms differ from when they form a right angle. The same shoulder torque can produce different club acceleration at those configurations. Velocity-dependent terms also grow with angular speed. A late release window emerges from the moving state, not from a universal clock time.

This helps explain why early positive wrist torque can be an inefficient tool in some optimized double-pendulum studies. It opens the relative angle before the system reaches a geometry where distal rotation produces a large clubhead-speed benefit. The result is model- and objective-dependent, but the mechanism is clear enough to test.

7.2.3 The Club Pushes Back

Action-reaction runs both ways. Accelerating the arm affects the club, and the club's inertia affects shoulder and wrist reactions. As the club swings outward, it can slow the arm or demand additional proximal torque. A description that calls the club "passive" must not imply it is dynamically invisible.

The wrist reaction contains radial and tangential components. Radial force keeps the club attached to the moving hand and supports curved motion. Tangential force contributes more directly to angular acceleration about the hand. Their signs and powers change through the downswing.

7.2.4 A Limiting-Case Test

If the club mass tends toward zero, its reaction on the arm should vanish. If the club length tends toward zero, its distal leverage disappears. If the links are decoupled, off-diagonal mass terms vanish and proximal torque no longer accelerates the club coordinate through inertial coupling. These limiting cases are more than classroom exercises; they are implementation tests.

Likewise, setting direct wrist torque to zero does not freeze the wrist angle. The unactuated coordinate can move because coupling and gravity remain. An underactuated system is not an unmoving system.

7.2.5 Why the Model Remains Valuable

A model earns its place by isolating a mechanism, not by resembling every surface detail. The double pendulum lets us compute exact pointwise contributions, integrate counterfactual branches, close work-energy balances, and sweep timing without the identifiability burden of hundreds of muscles. If a proposed mechanism fails here, adding anatomical detail will not rescue its simplest claim. If it succeeds, higher tiers must test whether it survives added degrees of freedom and measured constraints.

Model Result: The declared double-pendulum schedules reproduce a familiar finding: late distal torque can improve club speed, while early drive can reduce the favorable release window. Zero-torque branches still show substantial coupled club motion.

Human Evidence: The model's arm and club coordinates are abstractions. Human golfers show multi-planar shoulder, elbow, forearm, wrist, trunk, and lower-body motion. The double pendulum is not an anatomical fit, although it has a long and useful history as a mechanism model (Jorgensen 1999; Sharp 2009; McPhee 2022).

Practical Interpretation: The club can be carried before it spins rapidly relative to the hands. Delayed visible release need not mean delayed energy input; it may reflect when stored and transported energy becomes distal speed.

7.2.6 Building the Model One Assumption at a Time

The single pendulum begins with one angle, angular velocity, inertia, gravity, and perhaps a driving torque. It already teaches an essential lesson: torque, acceleration, and velocity are different. The same torque produces different acceleration as inertia or configuration changes, and speed reflects everything that happened earlier.

Add a second link and the state doubles. The club angle can be absolute or relative to the arm; either convention works if equations and signs are consistent. Kinetic energy now contains cross terms because both coordinate rates contribute to distal-mass motion. Those terms change with the angle between links. The club therefore responds even when its own applied torque is zero.

The fixed base is deliberate. It makes coupling visible without pelvis translation, balance, or ground contact. Lengths, masses, centers of mass, and inertias are declared rather than inferred from one golfer. The model's value comes from traceable causal experiments, not from looking anatomically complete.

7.2.7 A Slow-Motion Downswing

At downswing start, proximal torque accelerates the first link. The club initially lags because inertia and geometry resist immediate relative rotation. The wrist connection transmits force, so the club is already moving through space before a large release. This carried motion is lost if the swing is described only by the club's relative angle.

As arm angular velocity increases, velocity-dependent interaction terms grow. Changing arm–club angle changes effective coupling. Later, relative club rotation can accelerate rapidly. Depending on schedule, distal torque can add positive work, regulate release, or temporarily oppose it. There is no single instant at which stored energy is handed over like an object; mechanisms overlap.

Near impact, available time is short. A command with a physiological or actuator rise time may arrive too late even if its ideal step version looks effective. An earlier command can alter geometry so as to reduce later transfer. Preferred onset in this model is consequently a state-and-dynamics problem, not simply "as early as possible" or "as late as possible."

7.2.8 Tests That Make a Toy Model Useful

The mass matrix should remain symmetric and positive definite across the tested range. Zero-input conservative trials should maintain mechanical energy within declared numerical tolerance. Reducing timestep should converge outcomes. Independent formulations—for example Lagrangian and constrained Cartesian or spatial implementations—should agree. Known limiting cases should be recovered when a mass, link, or coupling is removed.

Then come causal controls. At matched state, remove proximal torque, remove distal torque, set velocities to zero for a pointwise velocity counterfactual, reverse relevant geometry, and integrate killswitch branches. If the claimed interaction survives only one plotting convention or one solver, it cannot support a broader argument.

7.2.9 What Carries Into Higher-Order Models

An exact numerical optimum rarely survives. A signed mechanism can: proximal acceleration changes distal reaction; geometry changes its projection; a late opposing moment can coexist with later speed benefit; and force allocation can be hidden by a net wrench. These become invariants to test on a moving base, in three dimensions, with shaft flexibility, and under two-hand constraints.

This ladder avoids opposite mistakes. One is rejecting a simple model because it lacks anatomy even though it isolates a real mechanism. The other is treating that mechanism as proof of a human strategy. Each rung asks which conclusion remains and which was an artifact of earlier assumptions.

7.2.10 A Reader's Replication Exercise

Choose one parameter set and torque schedule. Record the complete initial state, coordinate definitions, solver, timestep, tolerances, and output window. Reproduce baseline, then halve timestep. Set distal torque to zero only at a copied state and compare instantaneous acceleration with a short forward branch. Finally perturb mass and length. If the sign changes easily, the honest result is a sensitivity map rather than a law.

7.2.11 Reader Lab: Discover Coupling Yourself

Begin with both links hanging and zero velocity. Apply a brief proximal pulse while distal torque stays zero. Plot both angles, speeds, wrist reaction, and energies. Repeat with the club initially aligned, lagged, and partly released. The distal response changes because geometry changes even though its motor input does not.

Next copy one moving state and set all torques to zero. The links continue and exchange energy under inertia and gravity. Set velocities to zero in a separate pointwise calculation while retaining angles. The difference isolates the velocity-dependent part at that instant. Do not forward-integrate the artificial zero-velocity state as though a golfer stopped instantaneously; it is an attribution construct.

Finally, sweep distal-torque onset and amplitude. Plot a surface rather than one best trace. Add finite rise time and mass uncertainty. Mark regions where speed improves, where face/path proxy degrades, and where the result reverses. The exercise turns "delayed release" from a slogan into a conditional map.

7.2.12 The Most Important Limiting Cases

If distal mass tends to zero, its reaction on the arm should diminish. If the second link length tends to zero, the model approaches a different one-link inertia. If both links lock, they behave like one compound rigid body. If gravity is removed and no work enters, energy should remain constant. If initial velocity and inputs are zero, nothing should spontaneously accelerate except under gravity. These checks are simple enough that failure cannot be excused by biological complexity.

7.2.13 What a Lay Reader Can Safely Carry Forward

The useful insight is that distal motion depends on the entire linked state, not only distal effort. Early proximal motion can create conditions for rapid later club motion, and direct distal action can regulate that evolution. The model does not tell a golfer which muscle to activate or prove one delay is ideal. It teaches what measurements and counterfactuals a stronger model must preserve.

7.2.14 A Bridge to the Rest of the Book

The remaining models do not discard the double pendulum. They ask whether its coupling story survives separated hands, a moving base, spatial motion, and a flexible shaft. Whenever a later result looks complicated, returning to the two-link state, force, power, and counterfactual definitions provides a useful check. Complexity should enrich the mechanism without changing the meanings of its basic observables.

The checkpoint is simple: if a claim cannot be reproduced in the transparent two-link benchmark when that benchmark contains the claimed ingredients, debug it before adding anatomy.

7.3 Where the Picture Breaks

The two-link picture encourages the "whip" metaphor. A whip is flexible, distributed, and driven through waves; a rigid double pendulum has discrete links and joints. Both show distal speed amplification, but the mechanisms are not identical. Golf shafts are flexible enough to matter yet far from cloth whips.

The model also collapses two arms and two wrists into one connection, fixes a plane, and often prescribes the proximal base. It cannot identify scapular muscle action, grip-force allocation, clubface control, or injury load. These questions require later tiers.

Go Deeper: Read the monograph's double-pendulum mechanics, numerical mechanism audit, and Putnam's linked-segment synthesis (Putnam 1993).

Chapter 8

The Summation-of-Speed Story and Its Limits

8.1 A Concrete Picture

Walk forward inside a moving train. Relative to the carriage you may move at 2 m/s; the carriage moves at 20 m/s relative to the ground; your ground speed is roughly 22 m/s when directions align. Endpoint speed in a linked chain has a similar geometric addition. The clubhead moves because the hands translate, the club rotates about them, and all upstream joints contribute to the hand motion.

This gives the summation-of-speed story its appeal. Proximal motion carries the distal base, and distal rotation adds speed farther from the center. But unlike the straight train aisle, the velocity vectors rotate, segment lengths matter, and the same motions can cancel when directions oppose.

8.2 How the Mechanism Works

For a planar arm and club, clubhead velocity can be written as hand velocity plus velocity from club rotation about the hand:

$$\mathbf{v}_{head} = \mathbf{v}_{hand} + \omega_{club} \times \mathbf{r}_{head/hand}.$$

Hand velocity already contains contributions from proximal joint rates. This is a kinematic identity. It tells how motions combine at an instant, not which forces caused those motions or where their energy came from.

8.2.1 Why Distal Rotation Has Large Leverage

Linear speed from rotation grows with radius: $v = r\omega$. A given angular velocity near the club can contribute strongly at the clubhead because the radius is long. Yet creating that angular velocity requires energy and control under the system's inertia. The benefit of distal speed is not free leverage; it is a mapping between angular and endpoint motion.

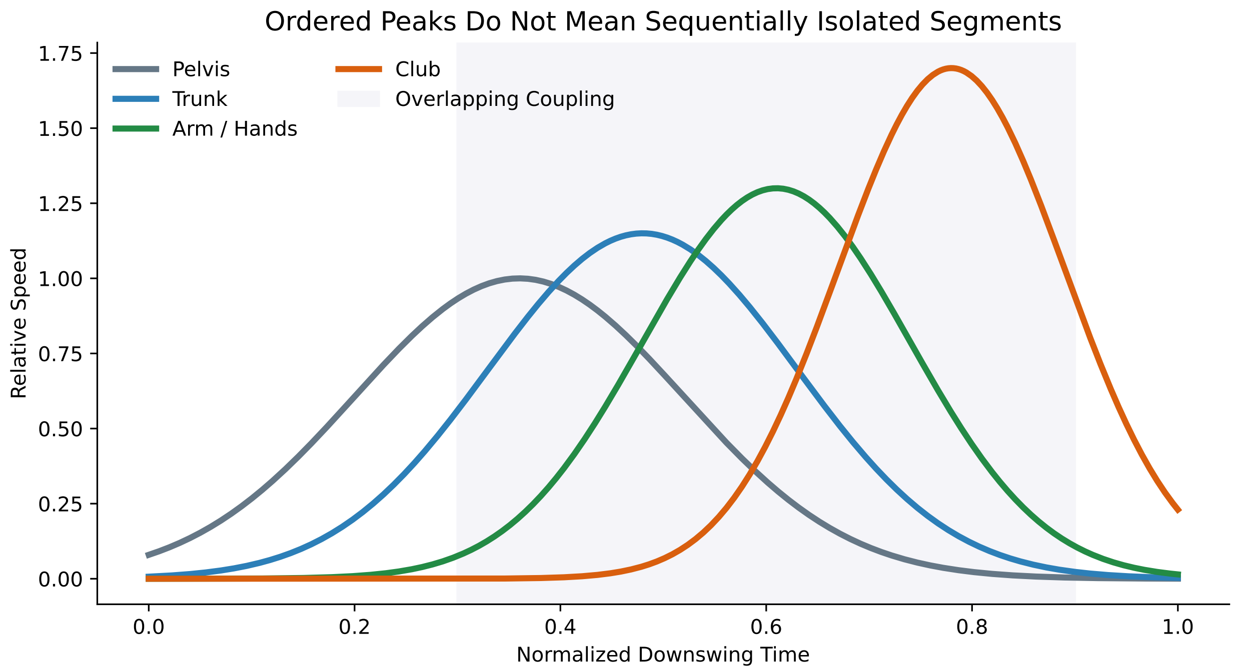

Figure 8.1: Segment-Speed Peaks Occur in an Ordered Sequence While Their Contributions to Clubhead Velocity Overlap in Time.

Timing matters because velocity vectors can align or oppose. If hand velocity and club-relative velocity point similarly near impact, their magnitudes add. If the club releases too early, its relative velocity may point less favorably later or change hand dynamics through reaction. The optimal schedule depends on the full trajectory and outcome definition.

8.2.2 Peaks Do Not Hand Off Like Relay Batons

When pelvis speed peaks and falls before trunk speed peaks, it is tempting to say pelvis speed was transferred into trunk speed. But speed is not conserved. Momentum and energy have conservation laws under stated conditions; angular speed does not. A declining pelvis speed may coexist with continued positive hip power, changing effective inertia, or work transferred through several joints.

Peak order is therefore a coordination descriptor. To claim transfer, pair it with joint powers, interface powers, segment energies, and external forces. Even then, the partition must be declared.

8.2.3 Sequence Can Vary While Outcome Remains Similar

Human movement is redundant. Different joint-rate combinations can produce a similar clubhead path and speed. One golfer may use more pelvis rotation and another more thoracic or arm motion. Averages can show a common sequence while hiding participant-specific solutions.

The uncertainty study reinforces this point in model space. Eight preselected control programs are all nondominated across held-out outcomes: improving one metric costs another. A single “ideal sequence” does not emerge from speed alone.

8.2.4 Phase-by-Phase Reading

During transition, the system changes direction and builds load. Early in the downswing, proximal motion can raise hand speed while the club remains lagged. In mid-downswing, geometry and velocity coupling change rapidly. Late, club rotation becomes a dominant contributor to clubhead speed. These labels are useful, but continuous powers and states are more informative than hard phase boxes.

A meaningful plot aligns events such as transition, peak hand speed, onset of rapid relative club rotation, and impact criterion. It then overlays segment speeds with the forces and powers proposed to cause them. The sequence becomes an entry point to mechanics rather than the conclusion.

Human Evidence: Proximal-to-distal peak ordering has been observed in many skilled striking and throwing tasks, including golf, but methods and definitions vary (Milburn 1982; Putnam 1991, 1993; Tinmark et al. 2010).

Model Result: The double-pendulum and higher tiers reproduce distal speed amplification through overlapping proximal transport, coupling, and late relative rotation. Their control programs are not unique human solutions.

Hypothesis: A useful strategy arranges geometry and state so the distal velocity contribution aligns with hand velocity near the chosen impact event without paying unacceptable face, path, load, or robustness costs.

8.2.5 Why the Relay-Race Story Is Attractive

In many skilled striking and throwing motions, proximal segment angular velocities peak before distal ones. Plot the curves and they resemble runners passing a baton. The picture captures the broad fact that distal speed is built on motion already present proximally. Problems begin when peak order becomes a mechanism by itself or a command to stop one segment before starting the next.

Endpoint velocity is a vector sum. Clubhead velocity includes proximal-base translation, contributions from trunk and arm rotation, relative club rotation, and deformation. These contributions overlap and change direction. A proximal angular velocity can decline while its current motion still contributes strongly to distal linear velocity. A distal segment can accelerate because of interaction forces before its own peak.

8.2.6 Peak Time Does Not Equal Transfer Time

A velocity peak occurs when that coordinate's acceleration crosses zero. An energy-transfer event is identified by power and work across a boundary. They need not coincide. Proximal speed can peak because distal reaction loads it, because drive declines, or because geometry changes—not necessarily because energy was deliberately "passed." Distal speed may keep rising after one joint- power term turns negative because other channels remain positive.

Filtering and coordinates can move peaks. Global segment angular velocity, relative joint angular velocity, one anatomical component, and the magnitude of a three-dimensional vector produce different event times. A sequence claim must state quantity, frame, filter, event definition, and uncertainty.

8.2.7 A Three-Link Thought Experiment

Imagine pelvis, arm, and club speed peaks at 300, 220, and 80 milliseconds before impact. Ordering alone does not reveal whether pelvis work caused club acceleration. Inspect power: pelvis-to-trunk transfer may be positive over a broad interval; hand power may alternate; shaft return may peak late. If a filter shifts the pelvis peak while power integrals remain stable, the causal story should follow the ledgers rather than the prettiest order.

Hold final club speed fixed while altering torque schedule. A model can produce the same peak sequence through different work allocations. Conversely, two schedules can have different sequences but similar final energy. Sequencing is an observable constraint, not an identifying fingerprint.

8.2.8 Overlap Is the Rule

Muscles do not operate as isolated switches, and segments never become mechanically disconnected. Proximal and distal torques coexist. Constraint forces act bidirectionally. The nervous system may prepare stiffness before visible motion and regulate objectives simultaneously. A better phrase is "a wave of changing emphasis across a continuously coupled system."

This matters for the arm-versus-wrist proposal. In one extreme, arms establish the dominant handle wrench while wrists resist and regulate. In the opposite, wrists create more club moment while arm action supplies another allocation. Both can generate familiar-looking kinematics. Bilateral wrenches, power histories, and activation measures are needed to separate them.

8.2.9 How to Use Sequence Evidence Well

Combine sequence data with kinetics. Report segment and joint kinematics, external and grip wrenches, interface power, work by phase, and uncertainty in event timing. Test whether a mechanism predicts not only peak order but also force direction, couple sign, and changes under geometry or equipment manipulation.

For an individual, repeatability matters. A mean peak time with wide within- person variability may be a weak control target. A state relation—such as a force-direction change over a consistent arm–club-angle range—may generalize better across tempo. That possibility must be tested rather than assumed.

8.2.10 What Would Change the Interpretation

If a participant preserves club speed while systematically reversing expected peak order, the order is not necessary. If the order appears but predicted transfer powers and hand-force signs do not, it is insufficient evidence for the mechanism. If order changes with coordinate definition, conclusions should be framed at the level that remains invariant. These productive failures turn a slogan into a bounded empirical statement.

8.2.11 Reader Lab: Rebuild an Endpoint Velocity

Take a planar three-link chain and compute endpoint velocity as the sum of each joint-rate contribution through its Jacobian column. Draw each contribution as an arrow tip-to-tail. Some arrows can partially cancel even when every angular speed is large. Change posture without changing joint rates and watch the arrows rotate. Geometry is part of the summation.

Now plot each segment's angular speed and mark peaks. Compare those marks with endpoint speed, joint power, and cumulative work. They will not generally align. Repeat after applying a low-pass filter or switching from relative to absolute angular velocity. If the reported peak order changes, record that methodological sensitivity rather than choosing the preferred plot.

For a model intervention, remove proximal torque at a copied state while keeping current proximal velocity. Immediate club velocity is unchanged because state is unchanged, but acceleration and future trajectory differ. This shows why an existing proximal velocity contribution is not the same as continuing proximal power.

8.2.12 Alternative Explanations for the Same Sequence

A proximal peak can precede a distal peak because proximal drive declines, because distal reaction brakes the proximal segment, because joint geometry changes effective inertia, or because the chosen coordinate reaches an extremum. Several explanations can coexist. The sequence alone does not select among them.

Competing mechanisms make different predictions for forces, powers, and response to perturbation. A transfer explanation predicts particular interface-power and reaction patterns. A purely kinematic constraint may reproduce peak order without the same work exchange. Measuring and modeling those discriminators makes sequence evidence useful.

8.2.13 Responsible Practical Translation

A golfer need not consciously pause each body segment. Skilled motion can show overlapping peaks as the coupled system and control change emphasis. Practical analysis should avoid chasing exact population-average peak times. It can instead ask whether an individual's motion, wrench, and outcome are repeatable and whether a proposed change improves the full tradeoff under uncertainty.

8.2.14 What the Sequence Can Still Tell Us

After all these cautions, sequence is not discarded. It is a compact description of how motion emphasis changes, a useful check on model realism, and a way to locate phases for kinetic analysis. It becomes much stronger when paired with repeatability and mechanism predictions.

For example, a model that predicts a late negative couple should also predict where that interval lies relative to segment-speed peaks and arm–club state. Measured disagreement can refute or relocate the mechanism. Sequence is thus most useful as one layer in a synchronized evidence stack.

The responsible conclusion is neither "sequence proves transfer" nor "sequence does not matter." It is that peak order constrains candidate explanations but does not identify one. Forces, powers, counterfactuals, and uncertainty complete the test.

8.2.15 Chapter Checkpoint

The observed sequence is one set of constraints on an explanation. The stronger explanation also closes forces and energy, predicts what changes under an intervention, and survives alternate coordinate definitions. If those pieces are absent, describe peak order without assigning a unique cause.

This standard still leaves sequence plots valuable for communication. Their role is to orient the reader in time and state, then point toward the forces and powers that require deeper analysis.

8.3 Where the Picture Breaks

The train analogy adds velocities in a straight line and ignores reaction. In a swing, walking "inside the train" pushes back on the carriage. Segment rotations alter one another through inertia and constraints. Three-dimensional vectors can cancel even when scalar angular speeds look large.

The phrase summation of speed can also imply that more speed at every segment is always better. A proximal segment may need to slow or reorient so distal motion aligns. Peak magnitude, timing, and direction all matter, and the best pattern depends on the task.

Go Deeper: See the monograph's summation-of-speed discussion, empirical evidence chapter, and the review of kinematic heterogeneity (McPhee 2022).

Chapter 9

Force Direction Changes the Job

9.1 A Concrete Picture

Push a shopping cart forward and it speeds up. Push sideways and it turns. Push down on the handle and the wheels may grip more firmly without much forward acceleration. The force magnitude can be identical in all three cases; direction changes the job.

At the grip, a hand force can point along the club, across it in the swing plane, or out of that plane. Along-club force largely creates axial loading and curved- path support. Across-club force has a lever arm and tends to turn the club. An out-of-plane component can affect lie, face orientation, and spatial motion. As the club rotates, a force fixed in the room changes its components in the club frame. Direction is a moving relationship.

9.2 How the Mechanism Works

Power begins with projection. The dot product $\mathbf{F} \cdot \mathbf{v}$ selects the component of force along point velocity. Moment begins with a cross product, $\mathbf{r} \times \mathbf{F}$, which selects the component perpendicular to the moment arm. The same vector can therefore do little translational work yet create a large moment, or deliver power through a moving base while its moment about a particular point is small.

Define club-aligned radial and tangential unit vectors. The force components are $F_r = \mathbf{F} \cdot \hat{\mathbf{e}}_r$ and $F_t = \mathbf{F} \cdot \hat{\mathbf{e}}_t$. The tangential contribution to moment about the hand is approximately LF_t for club length L. The radial component supports centripetal acceleration and shaft tension. Neither label says whether power is positive; that requires the corresponding velocity.

9.2.1 Vectors Rotate While Labels Stay Put

Suppose the laboratory vertical force remains upward through a short interval. When the club is nearly horizontal, upward may be close to club-normal. When the club is nearly vertical, upward may be largely axial. A plot of global vertical force can remain positive while the club-tangential component crosses zero. Mechanistic claims should use the frame tied to the proposed job and provide the transform back to measured coordinates.

The technical figures draw vectors on model snapshots because time traces alone hide this geometry.

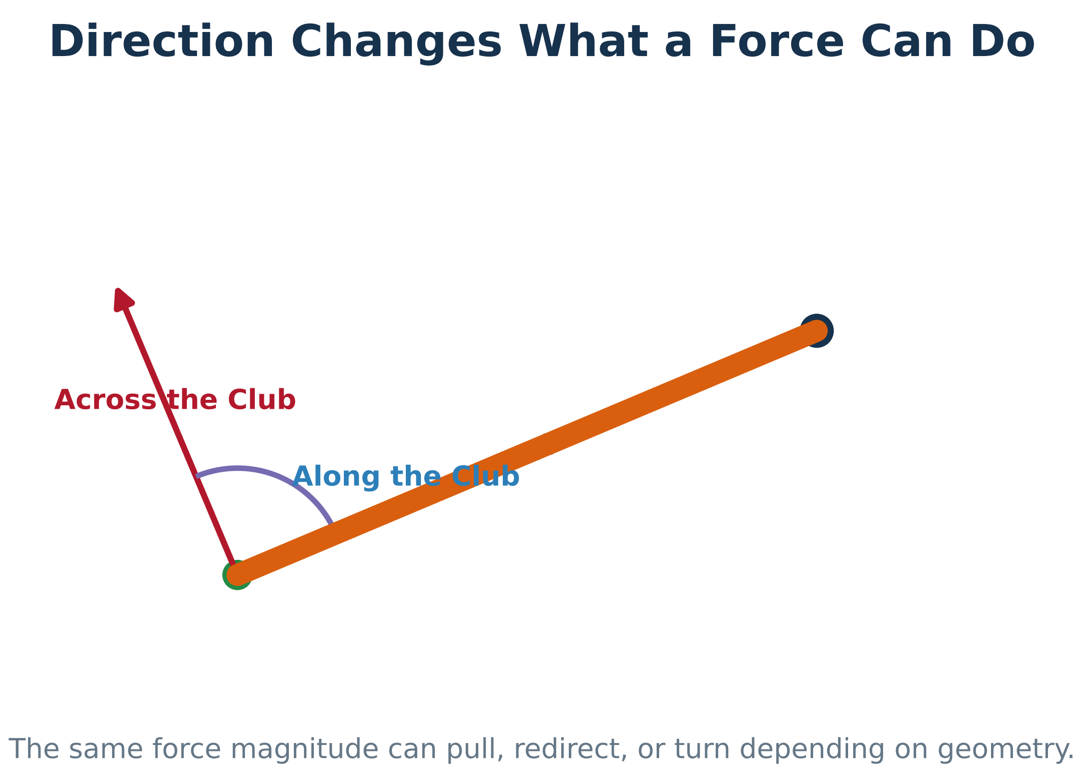

Figure 9.1: A Hand Force Is Split Into Along-Club and Across-Club Components With Different Mechanical Roles.

At each selected phase, the reader can see force origin, direction, scale, reference point, club orientation, and sign convention. A good vector figure is an argument, not decoration.

9.2.2 A Force Can Accelerate One Coordinate and Decelerate Another

Generalized coordinates are not independent Cartesian directions. Through the Jacobian transpose, one hand force maps into several joint torques. A force that creates positive club angular acceleration may create negative arm acceleration. This is precisely how interaction can redistribute motion without violating action-reaction.

The mass matrix then mixes generalized forces into accelerations. Looking only at the sign of F_t does not guarantee the sign of $\ddot{q}_2$ when other terms are present. Exact decomposition solves the coupled equations at the same state and reports the contribution rather than inferring it from one arrow.

9.2.3 Direction Through the Downswing

Early in the downswing, the hand path and club orientation can make a large reaction mostly radial. It carries and redirects the club. Later, as relative angle changes, a similar interaction can have a stronger tangential projection. Near the impact zone, two separated hand forces may form a negative swing-normal couple even while other pathways increase club kinetic energy.

This phase dependence is why the scientific question is not "is hand force good or bad?" It is: which component, in which frame, over which interval, about which reference point, with what power and impulse?

9.2.4 Three Names for Parts of One Motion Equation

The double-pendulum model gives a more precise vocabulary for the forces that appear as the arm turns and the wrist angle opens. It calls one contribution the **Coriolis Cross Term**. This part contains the arm's rotation rate multiplied by the wrist's relative opening rate. If either motion stops, the product is zero. It is therefore a compact way to ask how simultaneous proximal rotation and distal unfolding reshape the joint-coordinate balance.

A second contribution is the **Squared-Speed Term**. It contains one rotation rate squared. This is the coordinate-equation counterpart of the inward demand needed to keep mass following a curved path. In an inertial-frame free-body diagram, that inward requirement is called centripetal. A person describing the same balance from a rotating frame may use the outward inertial label centrifugal. They are two viewpoints on one interaction, not forces to add together.

Gravity supplies a third, configuration-dependent contribution. Unlike the other two, it does not require motion. It can help or oppose the declared coordinate depending on club orientation. Damping and applied moments complete the model's account. Adding every declared contribution reproduces the modeled acceleration; inventing a separate extra "centrifugal force" on top would count the same mechanism twice.

These categories are coordinate-dependent. In this study the arm angle is absolute and the wrist angle is relative to the arm. A different coordinate choice can move algebra between cross-speed and squared-speed bins without changing the physical trajectory. The complete velocity effect is the more stable object; the named split is an explanatory lens whose coordinate label must travel with every number.

9.2.5 Why an Outward Energy Story Needs More Than a Force Name

It is tempting to picture proximal energy as a parcel pushed outward by a Coriolis force. The safer picture is a state being prepared. Earlier applied moments and external forces build configuration and velocity. Once both links are moving, geometry redirects the consequences of that stored motion through the hinge. A Coriolis term can then make a large contribution even if the instantaneous distal command is small. It did not create energy from nothing; it describes how the moving system routes the effects of work already done and work still entering.

The sign also needs care. The code reports each left-hand equation term and its equal-and-opposite generalized drive. A positive equation term is not automatically positive acceleration, and a large force impulse is not automatically positive work. The drive must be paired with the appropriate velocity to obtain power. This is why the source display shows impulse and work side by side instead of giving one arrow a permanent "speed-producing" label.

9.2.6 Normalization Can Hide Magnitude

Force arrows are often normalized for visibility. That communicates direction but not magnitude. Other figures use one scale across all phases so arrow length can be compared. Reports should say which convention is used. A tiny vector and a huge vector drawn at equal length can otherwise invite a false mechanism.

Likewise, unit-body-mass normalization aids participant comparison but does not restore missing contact allocation. Directional averages can cancel when participants use different frames or handedness conventions. Preprocessing is part of the interpretation.

Model Result: The double-pendulum audit separates radial and tangential wrist reaction contributions and shows that their mechanical roles change with state. The two-hand geometry controls reverse or remove the predicted couple when the moment arms are reversed or collapsed.

Human Evidence: Instrumented grips and inverse dynamics can estimate force direction, but individual hand wrenches require direct measurement and careful frame registration and rigid-body convention (Featherstone 2008; Choi and Park 2020; Koike 2016).

Hypothesis: A late negative club couple can arise passively from the evolving projection of two-hand interaction forces, not solely from an actively commanded negative wrist moment.

Practical Interpretation: Replace "push harder" with a more scientific question: where does the force point relative to the club, hand path, and desired turning axis during that phase?

9.2.7 The Same Force Can Do Different Jobs

Draw a force arrow at the grip. If its point of application is moving in the same direction, the force delivers positive power. If motion is opposite, it absorbs power. If motion is perpendicular, instantaneous translational power is zero even though the force changes momentum direction. Rotate the club and the same laboratory-frame force obtains different radial and tangential components. No arrow has a permanent mechanical job independent of state.

This is why phrases such as "pulling on the handle" need coordinates. Pulling toward the golfer, along the shaft, along the hand path, or opposite clubhead motion are different vectors. A force can be inward relative to the clubhead path but tangential relative to a joint. The scientific treatment resolves every force into declared frames and also reports frame-invariant power.

9.2.8 Force at a Point Becomes a Wrench

A rigid body's external loading is summarized by a force and a moment at a reference point. Move the reference point and the force remains the same while the reported moment changes by the cross product of offset and force. This does not change the physics. It changes the coordinates used to describe the wrench.

With two hands, sum the two forces for net translation and sum each hand's moment about the chosen club point for net rotation. Equal and opposite forces can cancel in net force while retaining a couple. Forces in the same direction can add strongly yet create little couple if their combined line passes through the reference point. Both common and differential hand modes matter.

9.2.9 Following a Vector Through the Downswing

Early in the downswing, a large grip reaction may point largely along the club, maintaining the curved path and transmitting proximal motion. Its direct tangential acceleration contribution can be modest. As the club changes orientation, the projection can rotate even if the laboratory-frame force changes smoothly. Late in the downswing, a smaller force with greater tangential projection or moment arm may have a larger effect on club rotation.

At the same time, hand-point velocity changes. Positive grip force power can coexist with a negative swing-normal couple power because the translational and rotational pieces differ. A complete display therefore shows the force vector, moment vector, contact-point velocity, angular velocity, and their two power dot products. One scalar cannot represent this structure.

9.2.10 A Geometry-Control Experiment

Take a simulated state and its predicted bilateral hand forces. Run four reconstructions. In the first, retain original geometry. In the second, swap the hand contact positions but retain forces. In the third, reverse differential force direction. In the fourth, collapse hand separation toward zero. The claimed two-hand couple should respectively retain, reverse or otherwise change, reverse, and approach zero according to the declared cross products.

These controls are more informative than simply changing a force magnitude. They challenge the proposed geometric pathway. If the plotted couple stays unchanged when its moment arm is removed, the calculation or labeling is wrong. If the control behaves correctly, the model has passed a necessary mechanical test but has not shown that golfers use that allocation.

9.2.11 Measurement Frames Are Part of the Experiment

Instrumented grips report forces in sensor axes. Motion capture reports markers in laboratory axes. The club carries a moving frame, while anatomical analysis may use segment or joint frames. A calibration must locate each sensor and its orientation relative to the club. Time-varying transforms then put force, moment, position, and velocity into a common frame.

Small orientation errors can mix large axial forces into smaller transverse components. That matters when the proposed negative couple depends on a modest swing-normal projection. Calibration uncertainty should therefore be propagated into couple sign and timing, and reference transforms should be tested with known loads before any biological interpretation.

9.2.12 What a Useful Figure Should Show

A publication-grade force-vector figure should include coordinate axes, reference point, hand locations, arrow scale, moment-arm line, phase or timestamp, and sign convention. If arrows from different trials use automatic scaling, the caption must say so. A sequence of frames should retain scale where comparison matters. Accompanying plots should show components and resultant, not just one selected direction.

For reviewers, the underlying numeric table should be linked. That makes it possible to recalculate moments, transform frames, and verify that plotted arrows match reported values. Visual intuition is valuable precisely when it is anchored to auditable quantities.

9.2.13 Questions That Keep the Claim Honest

Where is the force applied? About which point is moment calculated? In which frame is "negative" defined? Is the vector measured, inferred, or simulated? Does it do positive or negative power at that instant? Does reversing geometry reverse the effect? Can another hand-force allocation produce the same net club wrench? Answering these questions turns an arrow from decoration into evidence.

9.2.14 Reader Lab: Verify a Two-Hand Diagram

Choose two three-dimensional hand positions and force vectors. Select a club reference point. Compute net force and moment directly. Translate the reference point and recompute; then verify total wrench power is unchanged when the new point velocity is transformed consistently. This catches the common mistake of moving the moment without moving the velocity term.

Resolve each force in laboratory, club, and swing frames. Reconstruct the original vector from every component set. Perturb sensor orientation by one or two degrees and observe how much a large axial force leaks into the smaller swing-normal component. That leakage becomes part of sign uncertainty.

Animate the arrows through a simulated late downswing using one fixed scale. Overlay moment arms and hand velocities. The visual should explain the calculated couple and power rather than merely make forces look dramatic. Release the numeric array so every arrow can be audited.

9.2.15 Competing Force-Direction Stories

One hypothesis may say inward handle force changes club acceleration through parametric geometry. Another may say a differential hand force supplies a negative couple. A third may say shaft reaction explains the observed wrench. They can coexist, but each predicts a different relationship among force direction, hand motion, deformation, and state.

Use partial interventions and measured mediators to separate them. Removing shaft flexibility in simulation tests the elastic route. Collapsing hand separation tests the differential-couple route.

Preserving force while changing handle path tests geometric projection. A good figure places these controls beside baseline so the reader sees how the mechanism can fail.

9.3 Where the Picture Breaks

The cart and planar arrows ignore distributed contact, three-dimensional axes, and deformable tissue. The real grip force acts across an area; a six-axis sensor reduces it to an equivalent wrench at a reference. That reduction preserves net effect but not pressure discomfort or local tissue load.

Force direction alone also cannot identify muscle intent. The same hand wrench can arise from many shoulder, elbow, wrist, and co-contraction patterns. EMG adds activation timing but not a unique force mapping.

Go Deeper: Use the monograph's force-vector mechanism chapter, coordinate force-source chapter, two-hand local geometry results, and the instrumented-grip studies (Choi and Park 2020; Koike 2016).

Chapter 10

Force Is Not Work Until Something Moves

10.1 A Concrete Picture

Hold a heavy suitcase motionless for a minute. Your muscles fatigue, but the suitcase has not moved and the external mechanical work on it is zero. Carry it horizontally at constant height and the upward hand force is nearly perpendicular to horizontal velocity, so that force again does little mechanical work on the suitcase. Lift it onto a shelf and force aligns with displacement; positive work raises gravitational potential energy.

This example feels unfair because biological effort is real in all three cases. That is exactly the lesson. Mechanical work, metabolic cost, muscle activation, and perceived effort are different quantities. A large grip force may stabilize the club with little club-level work. A modest force aligned with fast hand motion may transfer large power.

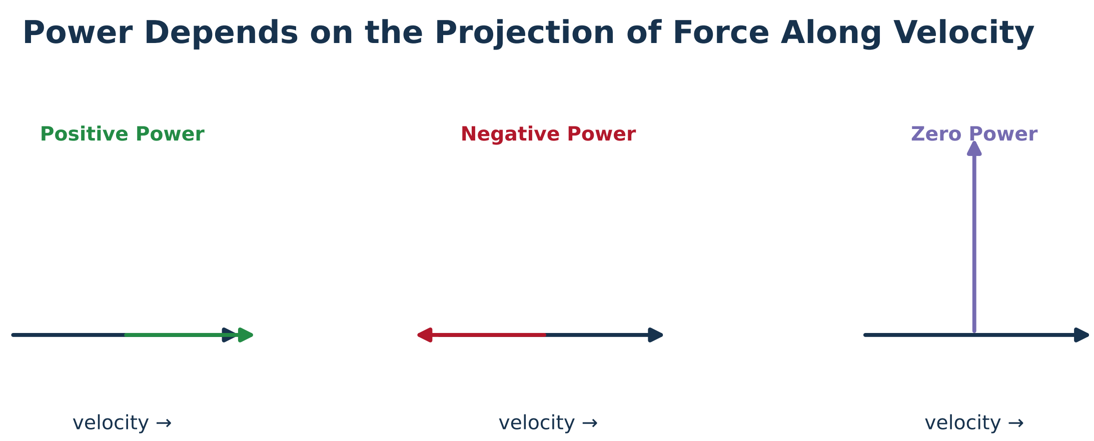

Figure 10.1: Force, Velocity, and Their Projection Distinguish Large Load From Large Mechanical Power.

10.2 How the Mechanism Works

Over a path, translational work is

$$W = \int \mathbf{F} \cdot d\mathbf{r}.$$

Over time, the same quantity is $W = \int \mathbf{F} \cdot \mathbf{v}\, dt$. Rotational work is $\int \tau \cdot d\theta$. Impulse instead integrates force over time, $\mathbf{J} = \int \mathbf{F}\, dt$. These answer different questions.

A force can have large impulse and small work when it acts mostly perpendicular to motion. It can have small impulse and meaningful work if it aligns with rapid motion over a short interval. Reporting magnitude, impulse, power, and work together exposes the difference.

10.2.1 Hand-Path Force Needs a Declared Path

Some golf analyses project net hand force along the hand path and integrate over path length. This can be a useful transfer observable. But a large value does not prove large biological effort. The force may be generated largely by drift and constraint reactions at the moving state, while active control contributes slightly against it.

The technical study evaluates this estimand across three model tiers. In two forward cases, drift supplies slightly more than 100% of signed hand-path force work because the contemporaneous control contribution is opposing. A percentage above 100 does not violate conservation; positive and negative contributions cancel to the total. In a two-arm local sweep, drift and control nearly cancel, making fraction language unstable.

10.2.2 What the Coriolis-Impulse Search Actually Optimized

The new pendulum example asks a deliberately narrow question: among 135 frozen torque programs, which one produces the greatest *absolute* along-hand-path impulse from the coordinate-defined Coriolis cross term before qualified impact? Absolute impulse counts both signs, so a force that reverses direction does not disappear through cancellation. Signed impulse is published beside it to retain direction.

Ninety-one programs reach the registered impact event. The maximum absolute Coriolis impulse is 13.817 N s with a 100 N m proximal drive, 10 N m of early wrist restraint, and wrist-drive onset at 0.100 s. That program reaches 46.780 m/s. The highest-speed program instead uses 5 N m of early restraint and a 0.125 s onset; it reaches 46.913 m/s with 13.701 N s of absolute Coriolis impulse. The optima are close, but they are not the same program. Maximizing a named force component is therefore not identical to maximizing clubhead speed, even inside this small deterministic model.

The signed values deepen the lesson. The maximum-absolute-impulse program has a signed Coriolis impulse of -6.825 N s and Coriolis generalized work of -100.69 J. A large absolute impulse can coexist with negative net work because the source reverses and because force, velocity, and generalized-coordinate signs matter. Squared-speed work is -33.73 J, gravity work is +25.56 J, and applied-drive work is +358.20 J in the same impact-truncated interval. These figures are an equation-term ledger, not a claim that a Coriolis agent supplied or removed a measured amount of biological energy.

10.2.3 The Hand-Path Arrow Does Not Carry the Whole Wrench

The modeled wrist point moves with the arm coordinate but not independently with the relative wrist coordinate. Its task Jacobian is therefore rank-deficient for mapping a two-coordinate generalized drive into one force-only hand-path vector. The software computes the minimum-norm force that can be represented at the wrist and keeps the leftover generalized couple as a residual. It never hides that remainder inside the displayed hand force.

For the two highlighted programs, the maximum retained residual is 54.44 and 52.81 N m. That is not a small numerical rounding error. It tells the reader that an along-path force plot answers a useful but incomplete question. A complete grip description needs a wrench, or an endpoint-and-couple mapping with enough independent coordinates and measurements.

At the top of the simulated swing, hand speed begins at zero, so no path tangent exists. The integration marks that interval undefined and uses only adjacent samples with a defined direction. More than 99.6% of the impact-truncated time is valid in the highlighted runs. Reporting that fraction prevents an arbitrary initial direction from contaminating the impulse.

10.2.4 Fractions Become Dangerous Near Cancellation

If drift contributes +10 J and control contributes −9 J, total signed work is +1 J. Calling drift "1,000% of total" is arithmetically correct and scientifically awkward. If total crosses zero, the fraction is undefined.

Better reports show signed contributions, absolute magnitudes, a cancellation index, and intervals separately. Ratios are reserved for denominators safely away from zero. This is not statistical fussiness; it prevents a small residual from turning two large opposing mechanisms into a sensational percentage.

10.2.5 Power Reveals Timing Hidden by Work

Two strategies can deliver the same integrated work with different power histories. One may load the shaft early and receive energy back late. Another may deliver direct power near impact. Their club energies at one instant can match while internal loads and robustness differ.

Plotting cumulative work beside instantaneous power is useful. The power curve shows when exchange occurs; the cumulative curve shows what remains after positive and negative intervals cancel. A phase-integrated table connects both to transition, early downswing, delivery, and impact.

10.2.6 Joint Power Is Not Muscle Power

Net joint power is often calculated as net joint moment times joint angular velocity. It is a legitimate mechanical quantity at the modeled joint. Muscles, tendons, ligaments, and co-contraction contribute to that net. A biarticular muscle can transfer energy between joints. Tendons can store and return energy. Therefore net positive joint power does not identify one shortening muscle, and near-zero joint power can coexist with substantial opposing muscle forces.

This distinction is central to the arm-versus-wrist proposal. A net club wrench can match while direct wrist moments, two-hand force couples, internal squeeze, and proximal joint loads vary. Club-level work is an outcome constraint, not a biological allocation measurement.

Model Result: The hand-path attribution separates pointwise drift and control contributions, then reports impulse, power, work, force vectors, joint effects, and cancellation rather than one percentage.

Human Evidence: Joint-work estimates depend on inverse-dynamics conventions and cannot uniquely recover muscle work. Instrumented contact and muscle-informed models can narrow but not eliminate ambiguity. Golf-specific work and energy studies still provide important system-level constraints (Winter 2009; Nesbit and Serrano 2005; MacKenzie et al. 2020).

Practical Interpretation: Force tells how strongly the system interacts; work tells how that force aligns with motion over distance; power tells how fast the exchange happens; effort belongs to a separate biological account.

10.2.7 From Force History to Mechanical Consequence

Start with a force vector through time. Its integral is impulse, which changes momentum. Pair each force sample with contact-point velocity and take the dot product to obtain power. Integrate power for work. Pair a moment with angular velocity for rotational power. These operations answer related but distinct questions; none can be safely inferred from the appearance of the force curve.

A high force applied while a point barely moves may create little work. A lower force acting through a long displacement can do more. A force perpendicular to velocity can bend a trajectory without changing kinetic energy at that instant. In a golf swing, all three patterns occur while the club follows a curved, rotating path.

10.2.8 The Hand Interface Has Two Power Channels

Choose a club reference point. Translational power is net force dotted with reference-point velocity. Rotational power is net moment dotted with angular velocity. Their sum is wrench power and is invariant to a consistent change of reference point. Reporting only the couple may miss positive translational power; reporting only net force may miss a large rotational exchange.

For two hands, individual contact powers can also be calculated. Their sum must match net wrench power after consistent transformations and inclusion of any contact moments. That closure is a strong audit. Yet individual hand power does not identify which muscle supplied it, because work can cross wrist, elbow, shoulder, and trunk pathways and because co-contraction can add internal cost without changing net external power.

10.2.9 A Late Negative-Couple Example

Suppose the net force supplies +900 watts of translational club power while the two-hand couple supplies -150 watts of rotational power over a short interval. Net hand power remains +750 watts. The negative couple is real and locally absorptive, but the hands as a whole still add club energy. If the couple changes orientation so its power later becomes positive, its integrated work may be near zero even while its impulse substantially changes angular momentum.

Now run a matched-state branch without that couple. The club may release sooner, changing geometry and reducing later positive power from another channel. The baseline could then finish faster despite the early negative work. That result would not mean negative work created energy; it would mean the intervention changed the route and timing by which other work acted.

10.2.10 Segment Work Is Not Muscle Work

Inverse dynamics produces net joint moments consistent with measured motion and external loads under a model. Multiplying net moment by joint angular velocity gives net joint power under the chosen convention. Several muscles, passive tissues, contact loads, and model residuals can contribute to that moment. Biarticular muscles can absorb energy at one joint and generate it at another. Assigning net joint work to one named muscle is therefore not warranted.

Musculoskeletal optimization can propose allocations, and EMG can constrain activation timing, but redundancy remains. A strong report keeps rigid-body joint work, muscle-tendon work, and metabolic energy on separate evidence levels. This distinction is especially important for claims about wrists "resisting passively" or scapular muscles "creating" a club couple.

10.2.11 Numerical Closure Before Storytelling

For a chosen system, the change in mechanical energy should equal integrated external power plus any declared dissipative or impact terms, within numerical residual. Calculate closure over the whole trial and important phases. Report absolute residual and residual relative to the effect being discussed. A two- joule residual may look small compared with total swing energy yet be fatal to a claimed one-joule mechanism.

Timestep refinement, alternate differentiation/filtering, and independent power calculations should preserve the conclusion. At contact interfaces, action-reaction and power consistency provide additional checks. A model that cannot close its ledger should not explain an unexpected sign by inventing a new transfer narrative.

10.2.12 What to Compare Across Strategies

For the arm-dominant and wrist-dominant extremes, compare more than terminal speed. Plot bilateral forces and moments; common and differential hand modes; interface power; cumulative work; club and body energy; shaft strain energy; joint load proxies; and face/path sensitivity. Align comparisons by both clock and state. Match the final club task when asking about internal allocation, and match the initial state when asking about causal input contribution.

A role reversal may preserve net club work while changing peak force, internal co-contraction, or timing sensitivity. Those are scientifically interesting differences even if clubhead speed barely changes. Conversely, a small speed gain may be unattractive if it depends on fragile timing or unmeasured internal load.

10.2.13 A Reviewer Can Recompute the Claim

The most useful release contains time, state, forces, moments, velocities, powers, cumulative works, energy terms, and residuals in machine-readable form. Units and frames live in a schema, and each figure links to its source. A reviewer can then verify dot products and integrals instead of accepting a caption. This is how a casual explanation remains connected to a scientific product.

10.2.14 Reader Lab: Recompute a Power Ledger

Download one time series of force, moment, reference-point linear velocity, and angular velocity. Check units and frames, then calculate translational power, rotational power, and their sum sample by sample. Numerically integrate each with at least two reasonable quadrature rules and compare. A difference larger than the reported mechanism needs explanation.

Next transform the wrench and velocity to another club point and verify summed power is invariant. Reverse the declared swing-normal axis and confirm only the coordinate label changes, not the physical power. Compare cumulative hand work with club energy change plus shaft, gravity, drag, and residual terms.

Finally, shorten and lengthen the integration window. If a claimed positive contribution depends entirely on an unreported boundary choice, the conclusion must name that interval. This exercise is simple enough for a reviewer and powerful enough to expose many sign and synchronization errors.

10.2.15 The Difference Between Necessary and Sufficient

Positive net work is necessary to increase a closed system's mechanical energy, but a particular positive joint-power peak may not be necessary because other routes can substitute. A negative couple may be sufficient to alter release in one model but not necessary for all high-speed trajectories. Counterfactuals and alternative allocations establish these logical distinctions.

Use cautious language: "contributed under this decomposition," "improved the declared outcome in this branch," or "was associated in this sample." Avoid "caused all transfer" unless interventions and closure exclude competing paths. The strength of the claim should match the design.

10.2.16 A Compact Reporting Template

Name the system, boundary, reference point, frame, sign, and interval. Report force and moment, point and angular velocity, power, cumulative work, energy change, residual, and uncertainty. State whether each quantity is measured, inferred, or simulated. Then link the machine-readable series. This template is short enough for every mechanism result and prevents most category errors.

10.3 Where the Picture Breaks

The suitcase treats the hand as one contact and the object as rigid. In golf, two hands, a flexible shaft, and moving joint centers complicate the boundary. Work at one reference must be transformed consistently, and distributed contact may store energy locally.

Mechanical work also does not judge usefulness. Negative work can be essential for control, accuracy, or preparing a favorable state. Positive work at the wrong time can worsen the final task.

Go Deeper: Read the full hand-path attribution chapter, coordinate force-source chapter, inspect its machine-readable evidence, and compare the segment energetics conventions (Robertson and Winter 1980; Winter 2009).

Chapter 11

Counterfactuals Are Scientific Scissors

11.1 A Concrete Picture

A baker wants to know what yeast contributed to a loaf. Removing yeast from the finished loaf is impossible. Baking a second loaf without yeast changes the rise, temperature history, moisture, and structure. Comparing one instant of the two doughs answers a different question from comparing the finished loaves.

Models let us make cleaner cuts, but each cut has a meaning. We can remove a torque at one frozen state and recompute acceleration. We can continue that altered branch forward. We can rerun the entire swing with a different schedule. Or we can remove gravity, damping, shaft flex, or a contact moment arm. These are not interchangeable "what ifs." They are scientific scissors with different blade shapes.

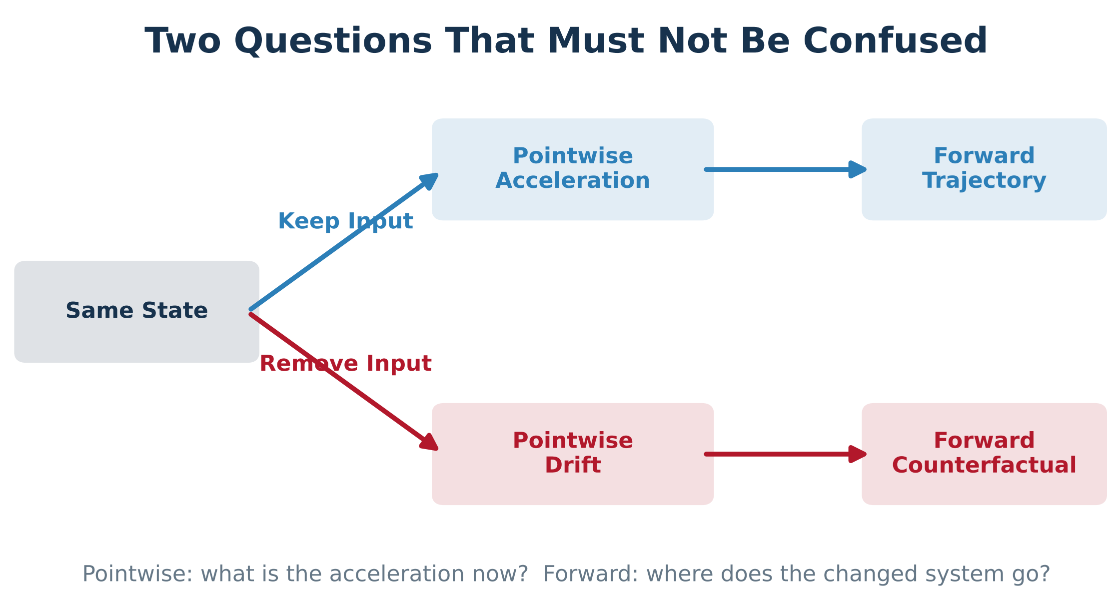

Figure 11.1: Four Counterfactual Branches Separate Instantaneous Attribution, Short-Horizon Persistence, Full Resimulation, and Mechanism Ablation.

11.2 How the Mechanism Works

A pointwise counterfactual holds configuration and velocity fixed, changes one input or term, and resolves acceleration or reaction force. Because the state is identical, the difference isolates an instantaneous modeled contribution. It does not say what a complete no-input trajectory would have looked like.

A matched-state forward counterfactual begins both branches at the same state and integrates with different future inputs. At the cut, attribution is clean. As time passes, states diverge. The difference now includes direct input effect, changed geometry, changed velocity coupling, and changed impact time. This answers whether the consequence persists over a declared horizon.

A full resimulation changes an input schedule from the beginning. It answers a strategy question: what trajectory and outcome follow if the entire command program changes? It cannot by itself attribute the outcome to one instant or mechanism because every later state changes.

An ablation removes a modeled mechanism such as gravity, damping, compliance, or contact separation. If the claimed effect remains when the proposed mechanism is absent, that mechanism is not necessary within the model. If it disappears, necessity is supported only for the declared family; another unmodeled mechanism could still produce a similar observation in humans.

11.2.1 Cut Time and Horizon Are Part of the Experiment

Choosing a cut after seeing the largest difference can exaggerate a result. The technical work therefore registers cut times, event definitions, and horizons. It repeats tests across nearby cuts and timesteps. A result that exists for one numerical sample but vanishes under refinement is not a mechanism.

Impact creates another trap. One branch may hit the impact criterion earlier. Comparing both at the baseline impact clock time may evaluate one branch after its own impact. Comparing each at its own first valid impact changes the state being compared. Both variants can be reported, but the distinction must be visible.

11.2.2 Decomposition and Intervention

An algebraic decomposition divides the acceleration at one state among gravity, velocity terms, controls, and constraints. An intervention modifies the model or input and observes the outcome. Decomposition says how the equation balances now; intervention says what changes under a defined alteration. Confusing them turns a bookkeeping term into a causal claim.

The zero-torque counterfactual force is a decomposition tool when computed at a fixed state. Integrating a zero-torque branch is an intervention. The technical paper reports both so readers can see where the instantaneous contribution continues and where trajectory divergence takes over.

11.2.3 Negative Controls Challenge the Story

Good experiments include cases where the mechanism should vanish or reverse. Coincident hand contacts remove a force-generated couple. Reversing moment arms should reverse its sign. Setting a dead zone to zero should remove any advantage attributed specifically to crossing that dead zone. Equalizing shaft stiffness should remove differences attributed only to flexibility.

Negative controls are more persuasive than a baseline-versus-favored comparison because they ask the mechanism to predict its own failure. A model that always produces the preferred result, regardless of geometry or removed mechanism, is describing an outcome rather than explaining it.

11.2.4 Counterfactuals Need Common Observables

Every branch should report the same quantities in the same frames: club speed, face/path proxy, forces, moments, powers, work, constraint residual, energy closure, and event timing. Otherwise one model can appear better merely because it reports a convenient metric.

This common-observable contract also connects model tiers. A double pendulum and a spatial contact model have different coordinates, but both can report an equivalent club wrench and interface power. Mechanisms that survive this ladder earn broader confidence.

Model Result: Pointwise wrist-force attribution, matched-state persistence, driver killswitches, geometry controls, zero-dead-zone controls, and stiffness ablations form a linked counterfactual program in the technical release.

Hypothesis: If delayed transfer depends on a particular preload or interaction mechanism, its advantage should disappear when that mechanism is removed while the task and state are appropriately matched.

Practical Interpretation: A fair "what if" must say what stayed identical, what changed, when the change occurred, and how long the altered branch was allowed to become a different motion.

11.2.5 Four Different Meanings of "Take It Away"

The phrase "remove wrist torque" can hide several experiments. A pointwise removal changes the torque only in the acceleration calculation at one copied state. A forward killswitch removes it from that state onward while other controls follow their original schedules. A reoptimized branch removes it but lets other controls compensate. A human perturbation asks a person to change a strategy and inevitably changes feedback, intent, and perhaps the starting state. These are not interchangeable.

Pointwise removal is best for decomposition. Forward removal is best for persistence within the declared model. Reoptimization asks whether another control route can substitute. Human manipulation tests whether the mechanism appears in an adaptive biological system. A complete research program uses the first three to sharpen predictions before attempting the fourth.

11.2.6 The Matched-State Rule

At the branch instant, copy every modeled state: positions, velocities, shaft modes, activation or transmission states, and controller memory. Change only the declared intervention. If one branch begins with a different club angle or force history, later divergence cannot be assigned cleanly to the removed input.

This rule is particularly important for preload. Comparing a continuously loaded backswing with a role-reversed downswing naturally produces different transition states. That comparison answers a strategy-level question but not the narrow causal question "what does the transmission state do?" A second matched-state experiment should equalize geometry and motion while manipulating only the declared gap or activation state. Both comparisons are informative when labeled correctly.

11.2.7 Pointwise Decomposition Does Not Add Like Percentages

At a fixed state, the acceleration equation may be decomposed into gravity, velocity, applied-control, external-load, and constraint contributions. These terms can oppose. One positive component can exceed total acceleration because another is negative. Dividing each by total and calling the result a percentage creates values below zero or above one hundred, which are mathematically possible but easy to misread.

Report signed components in physical units, cancellation indices, and residual closure instead. For nonlinear forward branches, effects generally do not add because each intervention changes the future state. A combined removal can differ from the sum of separate removals. That interaction is part of the result, not an accounting error.

11.2.8 A Counterfactual Sequence for the Negative-Couple Claim

Begin with baseline forward simulation. At selected late-downswing states, reconstruct bilateral contact and compute swing-normal couple. Next perform a pointwise removal of the relevant actuator or driver and observe the changed constraint reaction. Then run a short forward killswitch. Reverse hand geometry and force direction in separate negative controls. Finally repeat across model rungs and parameter samples.

The mechanism is strengthened if sign, timing, and geometric response remain coherent while magnitude varies. It is weakened if the couple persists when hand separation collapses, changes sign under a mere coordinate relabeling, or depends on one integration step. Human data must then test the predicted wrench, not merely reproduce the club trajectory.

11.2.9 Counterfactuals About Timing

To study onset, branch from several candidate states and apply identical command shapes shifted in state or time. Include finite rise time. Compare terminal speed, face/path proxies, peak loads, and sensitivity. The preferred command by one objective may be poor by another. Repeat under parameter uncertainty and on held-out samples.

A clock-shift experiment asks how a command schedule performs at different times. A state-trigger experiment asks whether activation at a geometric or dynamic event is more robust to tempo. These interventions can produce different answers. The study should expose both rather than translate one into a universal instruction.

11.2.10 Compensation Is a Separate Scientific Question

If wrist torque is removed and an optimizer increases arm torque to recover the same club motion, the result demonstrates redundancy. It does not show wrist torque was irrelevant in baseline. Compare internal loads, power routes, and robustness between baseline and compensated solutions. Multiple feasible routes are expected in a redundant biological system.

Conversely, failure to compensate within a restricted controller does not prove humans cannot. It establishes a limit of that control family. Expanding the family is a model-ladder step and should be preregistered enough to prevent the optimizer from becoming an explanation that can fit anything.

11.2.11 From Model Intervention to Human Falsifier

A safe human experiment cannot instantaneously remove a muscle torque. It can manipulate grip spacing, club inertia or stiffness, instruction, or perturbation while measuring the predicted mediators. The model must say which observable should change and in what direction. If a grip-spacing reduction should shrink a differential couple, that is more testable than asking a participant to "use less wrist."

The final evidentiary bridge is therefore indirect but disciplined: model intervention, robust observable prediction, physical manipulation, synchronized measurement, and a predeclared result that counts against the mechanism.

11.2.12 Reader Lab: Build a Branch Manifest

For one intervention, write a small manifest containing source run, copied state hash, branch time, removed input, retained inputs, controller memory, solver, tolerance, horizon, and primary outcomes. Generate baseline and branch from that manifest. A third party should be able to confirm that no hidden state changed.

Plot zero difference at the branch instant for all states and the expected nonzero difference in acceleration or reaction. Then show divergence through time. Repeat with half timestep and a second engine. Store both successes and any failed closure. This turns a counterfactual from a narrative choice into a reproducible intervention.

Run the same design pointwise, forward, and with compensation. The pointwise effect identifies local contribution; forward effect identifies persistence; compensation identifies redundancy. If conclusions differ, report why rather than choosing the most favorable version.

11.2.13 Counterfactual Etiquette

Never describe an artificial zero-velocity calculation as a motion a golfer could perform. Never call a reoptimized trajectory "the effect of removal" without noting compensation. Never compare branches with different initial states as though only control changed. Never turn nonlinear component differences into percentages assumed to sum.

These rules do not make causal modeling timid. They allow strong statements within scope: "At this exact modeled state, removing this driver changed this reaction by this amount," or "Over this forward branch, retaining the negative couple produced this later outcome." Precision makes the result easier to challenge and reuse.

11.3 Where the Picture Breaks

The baking analogy suggests one ingredient can be removed without changing the meaning of the recipe. In nonlinear dynamics, interventions change the state trajectory, and mechanisms interact. Removing damping can alter phase and impact time, not just energy loss. Counterfactual design reduces ambiguity but does not make coupled systems separable.

Human counterfactuals are harder. A golfer cannot repeat an identical internal state while one muscle command is magically removed. Randomized cues, perturbations, and participant holdout approximate interventions but include adaptation and measurement noise. Model results should guide experimental design, not substitute for it.

Go Deeper: Read the matched-state counterfactual chapter, the same-state spatial killswitch, and the registered experimental negative controls.

Chapter 12

Two Hands Make One Visible Wrench

12.1 A Concrete Picture

Turn a steering wheel by pushing down with one hand and pulling up with the other. The two forces can nearly cancel as a net force on the wheel's center, yet together they create a clear turning effect. Now squeeze the wheel equally from both sides. The internal load in your hands rises while the wheel's motion may not change at all.

Two hands on a golf grip create the same distinction. The club responds to the combined **wrench**: net force plus net moment about a chosen point. The arms and wrists may experience very different individual forces while producing the same club wrench. Club motion alone cannot reveal the hidden allocation.

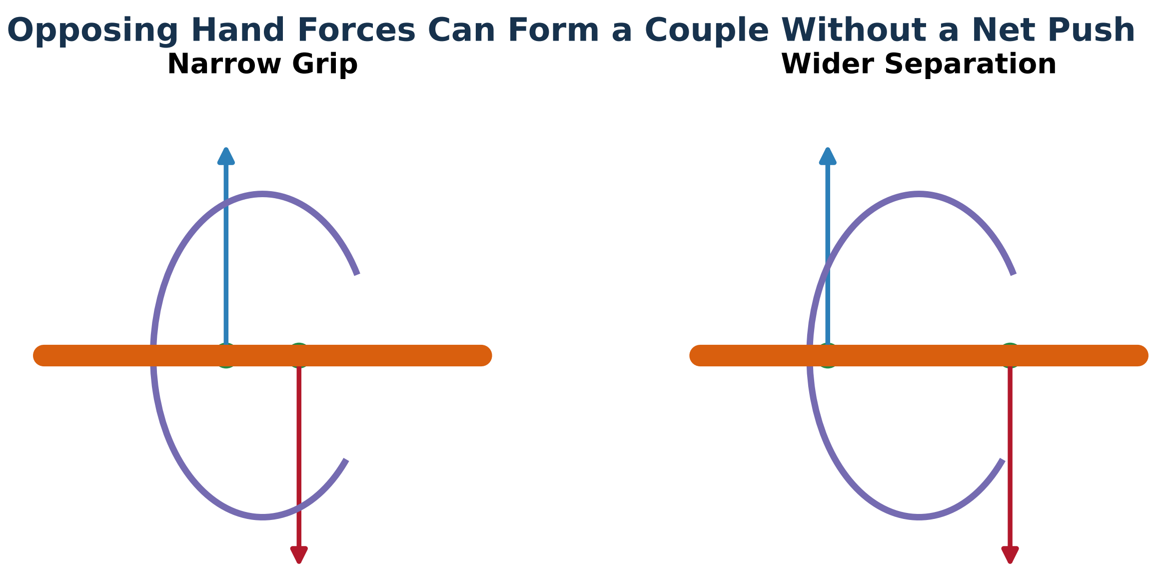

Figure 12.1: Equal and Opposite Hand Forces Produce a Turning Couple Whose Magnitude Grows With Grip Separation.

12.2 How the Mechanism Works

Let hand forces be $\mathbf{F}_L$ and $\mathbf{F}_R$ at positions $\mathbf{r}_L$ and $\mathbf{r}_R$ relative to club reference O. Include direct hand or wrist moments $\mathbf{M}_L$ and $\mathbf{M}_R$. The equivalent club wrench is

$$\mathbf{F}_O = \mathbf{F}_L + \mathbf{F}_R,$$

$$\mathbf{M}_O = \mathbf{M}_L + \mathbf{M}_R + \mathbf{r}_L \times \mathbf{F}_L + \mathbf{r}_R \times \mathbf{F}_R.$$

The club-level equations need this combined wrench. They do not uniquely need the four individual terms. Many allocations can produce the same sum.

12.2.1 Common and Differential Modes

It is helpful to reorganize the hand forces into a common mode and a differential mode. The common mode resembles their average and largely determines resultant force. The differential mode resembles half their difference and can create a couple or internal squeeze.

If we add $+\eta$ to one hand and $-\eta$ to the other, resultant force stays unchanged. Depending on contact geometry, moment may change or remain unchanged along certain null-space directions. This is the mathematical source of hidden internal loading.

12.2.2 A Wrench Is Observable; Allocation May Not Be

If club motion and inertia are known well enough, inverse dynamics can estimate the net external wrench required. It cannot decide whether a moment came from a direct wrist couple or separated hand forces unless contact is measured or an allocation criterion is imposed.

A minimum-norm allocation is a reproducible mathematical choice. It is not a measurement of how a golfer used the arms. A muscle-effort optimization is still model-dependent. Instrumented grips with force and moment sensing provide the most direct route to individual hand wrenches.

12.2.3 The Invisible Axial Push–Pull Mode

The ambiguity can now be stated exactly for two point forces. Six force components map to the six entries of net force and moment, yet the map has rank five, not six. Equal and opposite forces directed along the line between the hands produce neither net force nor net moment. That axial push–pull mode is therefore invisible to the net club wrench.

An independent measurement of that one internal axial scalar is sufficient to close the point-force rank gap in the idealized model. It is not sufficient if each hand can also apply its own three-dimensional contact moment: the full 12-to-6 bilateral-wrench map retains a six-dimensional invisible family. The calculation is a sensor-design result, not an anatomical result. It does not identify wrist action, scapular action, muscle recruitment, or intentionality, and calibrated noise, cross-talk, and moving contact centers still require separate qualification.

12.2.4 Internal Force Can Matter Without Moving the Club

Hidden squeeze may change grip stability, friction margin, joint compression, muscle activation, and effective stiffness. It can increase biological cost while leaving club-level work nearly unchanged. Conversely, a low-squeeze allocation may demand larger direct wrist moment. There is no single internal metric called "effort" that settles the trade.

This is why the technical article refuses to infer scapular recruitment from a club couple. A proximal generalized-torque subspace can be labeled and tested in the model, but the mapping to muscles, scapular motion, and hand pressure is not unique.

12.2.5 Action-Reaction Must Close

The force each hand applies to the club has an equal and opposite club force on the hand. A two-hand forward model solves both arms and club under closure constraints. The summed contact wrench on the club must equal the negative of the wrench returned to the body side after consistent transformation. Power and energy residuals provide further checks.

The spatial cross-engine test implements paired compliant contacts independently in MuJoCo and Pinocchio. Agreement on the common observables is stronger than repeating one code path, but it still validates a reduced mechanism rather than human anatomy.

12.2.6 Why Two Hands Can Create Negative Coupling

Near the late downswing, hand reactions need not be parallel. Their separated swing-normal components can produce a net couple whose sign opposes club rotation. This negative couple may coexist with positive club kinetic-energy growth because other powers and inertial terms remain. The model asks where the couple comes from geometrically and whether it survives forward integration.

Coincident-contact and reversed-moment-arm controls are decisive. Collapse the hand separation and the force-generated couple vanishes. Reverse the moment arms and its sign reverses. Those predictions distinguish a geometric mechanism from a plot-label artifact.

Model Result: The two-hand reconstructions close equivalent wrenches while revealing nonunique internal modes. A reduced spatial forward test retains a negative swing-normal contact couple for a declared late interval and passes the geometry controls.

Human Evidence: Bilateral grip sensors have demonstrated that individual forces can be measured, but the present model results are not measurements from golfers (Choi and Park 2020; Koike 2016).

Hypothesis: Opposed arm-chain actions can generate a useful club couple while wrists supply resistance or impedance, but only synchronized bilateral wrench and biological measurements can test that allocation.

Practical Interpretation: The club can feel one net push and twist while the two hands experience a much richer internal negotiation.

12.2.7 From Two Contacts to One Club Load

Each hand can apply three force components and three moment components at its contact reference frame. Transform both six-dimensional wrenches to one club frame, including the moment created by each position offset, then add them. The resultant club wrench determines rigid-body acceleration together with gravity, aerodynamics, shaft forces, and inertia. This sum is observable in principle, but its split between hands is not determined by club motion alone.

In a planar simplification, the arithmetic is easier: two two-dimensional forces at separated points create a net force and an out-of-plane moment. Even there, multiple pairs yield the same resultant. Adding equal and opposite force to the two hands changes internal loading while leaving net force unchanged; depending on direction, it can change the couple. In three dimensions the unobservable allocation space is larger.

12.2.8 Common and Differential Modes

Write each hand force as a common component plus or minus a differential component. The common mode tends to dominate net force. The differential mode often dominates the pure couple. This coordinate change does not claim the nervous system controls those modes explicitly. It is a clean way to see which parts of bilateral loading are visible at the club and which can cancel.

The distinction also clarifies "push–pull." Equal opposition is not required. One hand may supply a larger force, both may point partly in the same direction, and contact moments may matter. A measured push in grip-local coordinates can coexist with a laboratory-frame component that looks like a pull. The complete wrench should replace verbal labels wherever data permit.

12.2.9 How a Negative Couple Can Emerge

Near impact, proximal motion, club inertia, grip constraints, and contact geometry determine the reactions required at the hands. A differential force component acting through hand separation may project onto the swing-plane normal with negative sign even when neither hand is commanded with an explicit negative club torque. In a model this is a passive interaction contribution: it follows from state, drivers, and constraints.

That statement is narrower than "the scapulae passively torque the club." The generalized drivers can stand for many body actions. Reconstructing a negative contact couple does not uniquely map it to shoulder-girdle motion, wrist action, or muscle activation. The model supplies an observable prediction for bilateral wrenches; anatomy remains a separate inference problem.

12.2.10 A Worked Wrench Example

Let the hands be separated by 0.20 metres. Suppose they apply opposite 50-newton forces perpendicular to that separation. Net force is zero and couple magnitude is 10 newton-metres. Add a 100-newton common force and net force becomes 200 newtons while the ideal differential couple remains 10 newton-metres. If separation drops to 0.10 metres, the same opposing forces make 5 newton-metres.

Real force vectors are not perfectly perpendicular, contact points are distributed, and hand moments may contribute. The arithmetic nonetheless gives testable dependencies: couple scales with perpendicular separation and force; reversing either reverses sign; collapsing either collapses the couple. A model or measurement pipeline should pass those controls exactly within tolerance.

12.2.11 Matching the Club Task, Varying the Allocation

To compare arm-dominant and wrist-dominant strategies fairly, first specify a club-level task: perhaps the same net wrench history or a set of terminal club states. Solve for distinct internal allocations that meet it. Then compare joint demands, individual hand power, co-contraction proxy, transmission state, peak load, and sensitivity. The matched task prevents one strategy from “winning” merely because it was asked to produce a larger club moment.

Next relax the task and optimize performance under common constraints. This second experiment asks whether internal freedom can improve speed or robustness. Keeping matched-task and optimized comparisons separate avoids confusing mechanical equivalence with performance preference.

12.2.12 What Bilateral Sensors Add

Separate six-axis grip sensors directly constrain each contact wrench. Pressure mapping can reveal distributed contact and shifts in effective application point. Shaft strain helps close the club load balance. Synchronized motion supplies velocities for contact power and transforms sensor axes to club and laboratory frames. Force plates and body motion support whole-system closure.

Calibration needs known loads, crosstalk characterization, thermal drift checks, and frame registration. Grip modification can itself change behavior, so the instrumented club should be compared with a standard club. Uncertainty in hand position and sensor orientation should propagate to couple sign and magnitude.

12.2.13 Why Closure Is Not Calibration

A synthetic trajectory qualification makes the distinction quantitative. A net-wrench-only inverse can reproduce the net club wrench to numerical tolerance while missing the individual point-force allocation by 11.86 N RMSE and the invisible axial mode by 29.05 N RMSE. Closure therefore verifies the resultant, not the bilateral allocation.

The same controlled study separates practical measurement errors. Correcting a declared cross-talk matrix reduces allocation RMSE from 0.94 N to 0.15 N; a 10% residual calibration error raises it to 0.18 N. Holding contact centers fixed through an 8-mm migration causes 2.02 N RMSE, whereas tracking those centers removes that modeled bias. The combined registered case yields 1.02 N RMSE and 3.87 N 95th-percentile error over 32 seeded trials and 301 samples.

These values come from a synthetic point-force sensor qualification, not a physical transducer, distributed-pressure experiment, or participant study. They define pre-acquisition acceptance tests: verify the augmented rank, report allocation error in addition to wrench closure, calibrate cross-talk, track effective contact centers, propagate residual calibration uncertainty, and fail closed when any required channel or registration is absent. Governed bilateral six-axis human data are still required to test the proposed hand, wrist, arm, or scapular mechanisms.

12.2.14 Internal Force Is Not Wasted Force

An equal-and-opposite hand mode may cancel at the club’s center-of-mass force balance while increasing grip compression, joint load, or stiffness. It may help stabilize the clubface or may be needless co-contraction. Net club motion cannot decide. Outcomes and internal-load measurements are required.

This is where the question of "slack" becomes scientific. If continuous bilateral loading reduces a measurable force-development delay or perturbation response time without excessive internal cost, preload has operational value. If the same club task occurs with no gap or robustness difference, the proposed benefit is unsupported. A feeling of connection is not yet a measured state.

12.2.15 One Point per Hand Is Itself a Modeling Choice

Treating each hand as a single contact point is convenient, and the natural worry is that the convenience is doing the work. That worry has been tested directly.

Each hand was replaced by one, three, or five tension fibers spread across a 30 mm band, while **total** grip stiffness and damping were held fixed at 1800 N/m and 18 N·s/m. That control matters: adding contact points without holding the total constant would silently stiffen the grip, and any change could then be blamed on stiffness rather than on distribution.

Across 288 trajectories nested at 4, 10, 25, and 50 ms, the peak force on a single station reached 4.03 N, and the most loaded station carried **77% of the load** — distribution is not uniform, which is exactly why a single point can mislead about local loading. No contact opened during these runs. Going from three fibers to five changed the answer by 0.00355 at the fine timestep, so the description is converging rather than drifting with resolution.

What this earns is narrow and worth stating precisely: grip discretization can now be **compared fairly**, and the comparison is stable. It does not measure grip pressure, identify finger anatomy, or model friction or soft tissue. A number of stations is not a set of fingers.

12.2.16 Minimum Evidence for a Human Claim

The model prediction should specify phase, frame, couple sign, magnitude range, geometry dependence, and competing explanations. A human test should measure bilateral wrench, motion, shaft response, and relevant activation, with participant-held-out analysis. Only then can researchers ask whether a modeled contact mechanism is commonly used, individually variable, or absent. Until then, it remains a falsifiable mechanical hypothesis—not a prescription.

12.2.17 Reader Lab: Explore the Hidden Mode

Start with one pair of hand forces that produces a target club wrench. Add a small vector to one hand and subtract it from the other. Recompute the resultant. Depending on the vector and hand separation, net force stays fixed while moment changes. Add compensating contact moments so the full target wrench is restored. You have constructed a distinct internal allocation with identical rigid club loading.

Repeat while minimizing different costs: total force squared, peak wrist moment, lead-hand load, or deviation from a measured pressure pattern. Each criterion selects a different solution. This makes the role of regularization visible. Without bilateral measurement, none should be called the observed allocation.

Now add simulated sensor noise and contact-position uncertainty. Estimate the confidence interval on swing-normal couple and on each hand's contribution. A negative mean with an interval spanning zero is not firm evidence of sign. The lab links elementary wrench geometry to realistic qualification.

12.2.18 How This Guides Product Design

The software should display net wrench and the family of compatible allocations when individual forces are unavailable. When bilateral data exist, it should show measured allocation with uncertainty and closure residual. Any optimized allocation should be labeled as a model solution with its cost function.

This prevents the interface from converting hidden assumptions into anatomical facts. It also makes the tool useful for experiment design: users can see which sensor or manipulation would shrink the ambiguity most.

12.3 Where the Picture Breaks

The steering wheel is rigid, symmetric, and fixed to one axis. A golf grip moves in space, the shaft bends, wrist axes rotate, pressure is distributed, and both arms attach to a moving body. Two point contacts are a reduction.

The common/differential transformation also does not identify comfort or intent. The same internal wrench can arise from different muscles and co-contraction. Technique claims need participant-level evidence.

Go Deeper: Read the complete two-hand wrench mechanics, the forward constrained model, and the instrumented-grip literature (Choi and Park 2020; Koike 2016).

Chapter 13

Negative Torque Can Help a Positive Outcome

13.1 A Concrete Picture

When you throw a ball, muscles often brake one segment while another continues accelerating. When you drive through a bend, turning the steering wheel left can be correct even while the car's velocity still points partly right. Signs belong to axes and moments; words such as *helpful* belong to objectives.

In a golf model, positive club rotation is defined by a chosen swing-normal axis. A negative torque points the other way. It may reduce club angular speed, retain a relative angle, preload an interface, stabilize face orientation, or arise as a passive reaction. None of those interpretations follows from the minus sign alone.

13.2 How the Mechanism Works

In one rotational coordinate, power is $P = \tau\omega$. If the club rotates in the positive direction and torque is negative, power is negative: that channel removes club energy at that instant. But total club energy can still rise because hand-force power, shaft release, gravity, and inertial coupling contribute. A negative component in a sum need not make the sum negative.

13.2.1 Braking Can Prepare a Better State

An early opposing wrist command can retain the wrist-cock angle against opening. It may do negative work locally while allowing proximal work to carry the club into a geometry where later release has greater endpoint leverage. Whether the trade helps depends on later work, timing, actuator limits, and outcome metrics.

This resembles holding a spring-loaded door briefly before letting it open, but the club is not powered by one spring. Retention changes the entire coupled state. It can increase proximal demand and alter hand path. The benefit must be shown by forward simulation and work accounting, not assumed from the appearance of lag.

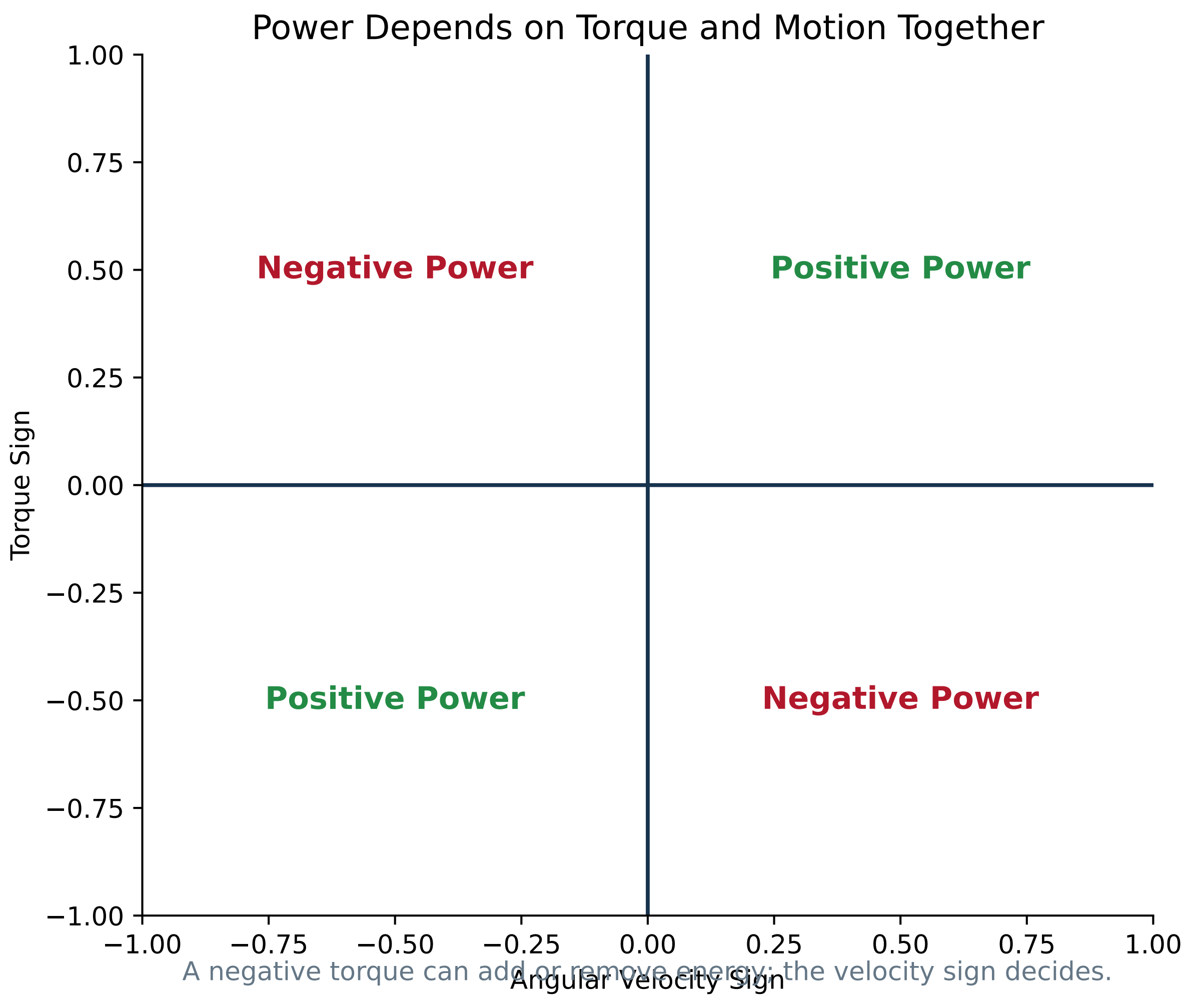

Figure 13.1: Torque and Angular-Velocity Signs Form Four Power Quadrants, Showing That Negative Torque Can Supply or Remove Energy.

13.2.2 A Passive Negative Couple

The WSCG-inspired two-hand question is different. It asks whether constraint and interaction forces can create a negative club couple even without commanding a matching direct negative wrist moment. In the reduced model, separated hand reactions project into a swing-normal couple whose late sign can be negative. That is a reaction mechanism, not proof of passive human wrists.

The technical work separates direct wrist moments from force-generated grip couple. It reconstructs the equivalent wrench and applies geometry controls. The negative couple persists in a declared forward interval, vanishes when the moment arm is collapsed, and reverses with reversed geometry.

13.2.3 Negative Power Is Not Lost by Definition

Negative work at one interface may transfer energy out of the club coordinate and into another body or elastic state. Damping turns energy into heat, but a conservative spring can return it. A negative wrist-power interval therefore needs a destination in the ledger.

At human joints, negative net power is often called absorption. Muscles may act eccentrically, tendons may store energy, or opposing muscle groups may coexist. Net joint power does not identify which tissue performed the work.

13.2.4 Timing Changes the Verdict

The same negative torque applied early, late, or after impact can have different effects. Early retention may preserve geometry; late braking may reduce speed; brief negative coupling near impact may coexist with a still-positive net club power. Strategy comparisons must report onset, duration, impulse, work, and the state at application.

The uncertainty experiment adds another caution. An early opposing command improves lower-tail speed relative to one late-drive program but worsens a planar face/path proxy. Both remain nondominated. “Helpful” requires a declared vector of outcomes.

Model Result: Negative direct or interaction torque can coexist with a positive final speed effect in declared schedules, but the mechanism and trade differ between retention, passive coupling, and robust-control tests.

Human Evidence: Wrist EMG and inverse dynamics show phase-dependent activity, but available measurements do not uniquely identify a passive two-hand negative couple or its muscle source (Morrison et al. 2016; Nesbit 2005).

Hypothesis: Late negative club torque may be produced partly by two-hand interaction geometry while energy continues arriving through proximal and elastic pathways.

Practical Interpretation: Read a torque sign as an arrow, not a grade. Ask which axis, which velocity, which power, which phase, and which final objective.

13.2.5 First Decide What “Negative” Means

A sign has meaning only after a coordinate is declared. In a planar model, positive club rotation may be defined in the downswing direction relative to the arm. In a spatial model, “negative swing-normal moment" refers to projection onto a time-varying axis. An anatomical flexion moment, a club-frame moment, and a generalized coordinate torque are not automatically the same quantity.

Next distinguish torque from power. A negative torque acting while angular velocity is positive has negative power. It removes energy through that coordinate at that instant. A negative interaction contribution to generalized acceleration is different again: it describes how state-dependent dynamics affect motion under a decomposition. Careless writing collapses these meanings and makes an ordinary sign look paradoxical.

13.2.6 Braking Can Improve a Later Outcome Without Creating Energy

Consider steering a loaded cart around a corner. A brief braking or inward force can reduce one velocity component, improve orientation, and permit more useful acceleration afterward. The brake does not supply energy. It changes state so other forces act more effectively. In a linked swing, an opposing distal moment can delay relative release, alter coupling geometry, or regulate face/path while proximal and hand forces continue to do work.

A matched-state simulation can make that precise. Compare baseline with a branch where the opposing moment is removed. If the branch releases earlier and ends slower, baseline's negative interval contributed to the later outcome through state evolution. The energy ledger must still show where positive work came from. Language such as "negative torque creates speed" should be replaced by "the schedule containing negative torque produced a faster later state through the following measured pathway."

13.2.7 Direct, Interaction, and Contact Contributions

Direct generalized torque is an input term specified by the controller. Interaction torque is a decomposition of dynamics arising from configuration, velocity, inertia, and other driven coordinates. Contact couple is reconstructed from forces and moments at hand interfaces. All three can be negative over the same interval, but they are not synonyms.

A passive negative contact contribution is especially easy to overstate. It means that, given the modeled state and drivers, enforcing contact produces a couple with negative projection. Muscles are still active elsewhere; maintaining geometry and stiffness may require work. "Passive" describes the absence of an explicit torque in that selected model channel, not the absence of biological effort.

13.2.8 The Role-Reversal Spectrum

At one extreme, arm and shoulder-girdle action establishes most of the handle wrench while wrists maintain alignment, stiffness, or a smaller regulating moment. Continuous loading from backswing through early downswing may keep a modeled transmission variable engaged. At the other extreme, wrists establish more of the backswing club moment while arm action resists; during downswing the allocation reverses, with wrists opposing while arms drive.

These extremes organize experiments; real golfers can occupy mixtures that change by phase. To compare them, match club-level task and initial state where appropriate. Measure sign changes in bilateral wrench, activation timing, force-rise delay, stiffness proxy, shaft response, and resulting work. A strategy name cannot substitute for these observables.

13.2.9 When Retention Is Useful

Retention is valuable only relative to an objective. Delaying release can allow proximal motion and geometry to develop before rapid relative club rotation. It may preserve a later positive-power window or improve face/path regulation. Too much or too-late retention can leave insufficient time to accelerate, increase load, or deliver poor impact orientation.

This produces an optimum region rather than a maxim: some opposing contribution, over some state interval, under some parameter distribution. The region shifts with club inertia, shaft stiffness, tempo, strength, and objective weights. A model should map that region and its uncertainty, not canonize one torque curve.

13.2.10 A Simple Schedule Comparison

Imagine three commands applied from a copied early-downswing state. Schedule A uses positive distal torque throughout. Schedule B uses a short negative phase then positive drive. Schedule C keeps the negative phase too long. A plausible model can make B fastest, A earlier-releasing, and C too late. But change arm speed, inertia, or rise time and the ranking can change.

The scientific result is not that B is universally best. It is the response surface and mechanism: when B wins, which power channels and geometry explain it; where it loses; and how readily the ranking reverses. Held-out parameter samples and independent solvers test whether that explanation is robust.

13.2.11 Human Evidence Needed

Kinematics can show relative release and angular acceleration. Inverse dynamics can estimate net generalized moments with assumptions. Bilateral grip sensors can measure the contact couple. Shaft strain can show concurrent storage and return. EMG can show activation timing, but force and passive tissue contribution remain uncertain. Perturbation or system identification may estimate effective stiffness and force-development delay.

A convincing dataset synchronizes these signals and preregisters the late interval and sign conventions. It tests whether predicted negative couple precedes or accompanies the expected state change, whether geometry manipulations alter it, and whether the effect generalizes across held-out participants. A motion trace alone cannot establish passivity.

13.2.12 Failure Conditions

The proposed benefit is weakened if an opposing interval never improves any declared speed–control tradeoff; if apparent benefit disappears after matching initial state; if energy accounting assigns positive work to the negative channel through a sign error; if contact couple does not respond to geometry; or if human wrenches show the opposite sign in the preregistered window. Each failure would refine the theory rather than threaten the value of testing it.

13.2.13 Reader Lab: Audit a "Negative Helps" Claim

Locate the coordinate and sign definition. Plot torque, angular velocity, power, and cumulative work for the same channel. Mark the proposed negative interval. Next plot all other power ports and total energy residual. Identify the positive work source supporting any later speed increase.

Copy state at interval start and remove only the negative input or contact mode. Compare immediate acceleration and forward outcomes. Repeat after matching terminal task through reoptimization. The first comparison asks causal contribution; the second asks whether another route substitutes. They should not be blended.

Reverse geometric controls, alter interval boundaries, and sample parameters. If benefit is robust only for one hand-picked trace, report a demonstration. If the mechanism retains sign and improves a defined tradeoff over a broad region, report a conditional strategy result. Neither supports an anatomical instruction without human measurements.

13.2.14 Three Statements That Sound Similar but Are Not

"The wrist moment was negative" is a kinematic/dynamic sign statement. "The wrist removed club energy" is a power/work statement requiring angular velocity and integration. "The negative wrist action improved final speed" is a causal counterfactual statement. A paper should provide separate evidence for each.

Likewise, "passive negative torque" can mean a model interaction term, an elastic material reaction, or absence of measured activation. These meanings must be named. The project uses "passive" only with the producing model channel; it does not infer human intention or zero muscle work.

13.2.15 The Nonparadoxical Summary

Negative torque is ordinary signed mechanics. Its usefulness, if any, comes from how it changes state, timing, and other power pathways—not from producing energy with the wrong sign. Keeping that sentence visible removes the apparent paradox while preserving the interesting causal question.

13.3 Where the Picture Breaks

The brake metaphor suggests negative torque merely slows an independent wheel. In a coupled swing it also changes reactions, hand path, and future geometry. The passive label is especially dangerous: zero selected command in a model does not demonstrate relaxed muscles or absent neural control.

A planar sign also cannot represent all three-dimensional wrist and clubface moments. A favorable swing-normal couple could coexist with an unfavorable face-twisting moment.

Go Deeper: See the passive couple-reversal chapter, the timing strategies, and the wrist-torque simulation literature (Sprigings and Neal 2000).

Chapter 14

Arms, Wrists, Preload, and the Meaning of Slack

14.1 A Concrete Picture

Turn a two-hand valve. One strategy uses the arms to create most of the opposing push-pull while the wrists remain firm. Another uses wrist moments more directly while the arms stabilize. The valve can receive the same net turning effect in both cases, but the loads inside the person differ.

Now reverse direction. A gear train already loaded against one tooth face can transmit the new magnitude immediately if load direction stays the same. If the load reverses across backlash, it moves through a gap before the opposite face engages. Biological tissue is not a gearbox, but the picture creates a testable question: does maintaining preload and load direction reduce a measurable transmission delay compared with complete role reversal?

14.2 How the Mechanism Works

The first experiment holds the club-level task fixed. At one state, proximal shoulder-elbow controls with zero direct wrist moment can produce the target club moment. Direct wrist controls with zero proximal torque can also produce it. Convex mixtures span the continuum. The club receives the same 8 N m task, yet modeled hand force and generalized-torque norm vary with allocation and club angle.

This establishes wrench equivalence with internal non-equivalence. It does not identify scapular retraction, muscle activation, or a preferred technique. The labels *arm channel* and *wrist channel* describe generalized actuator subspaces.

14.2.1 Two Extreme Stories

At one end, proximal arm-chain action creates the grip couple while wrists supply resistance or impedance. The same general proximal drive can continue across backswing and early downswing while signs remain loaded. At the other end, wrist moments create more of the backswing couple while arms resist; during downswing, roles reverse so arm-generated couple becomes positive and wrist action opposes release.

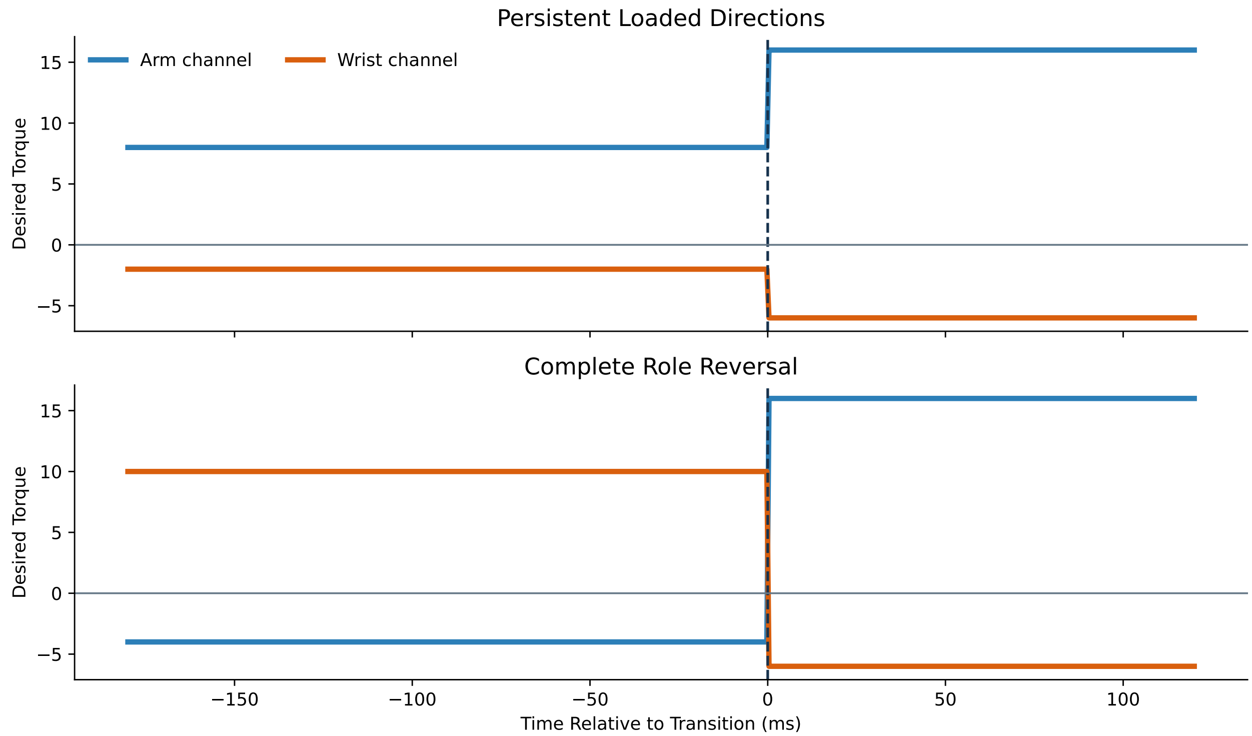

Figure 14.1: Persistent Arm Drive and Complete Arm-Wrist Role Reversal Cross a Command Transition With Different Modeled Transmission Histories.

These are useful poles, not a binary classification of golfers. Real allocations can lie between them and vary continuously through time. The comparison asks what each pole predicts under matched tasks.

14.2.2 Replace "Slack" With Measured Variables

Slack can mean tendon slack length, grip gap, low muscle activation, low tangent stiffness, joint laxity, series-elastic extension, or a delay in force rise. Those are not synonyms. The study chooses one operational model: a rotational dead zone followed by linear stiffness and first-order torque development.

An abstract channel transmits zero torque while deflection remains inside its declared dead zone. This lets the proposed gap mechanism be falsified. It does not claim wrists or scapulae contain literal backlash.

14.2.3 Continuous Preparation Matters

Starting a downswing simulation from a conveniently loaded equilibrium would hide how preload arose. The finite-history test begins both channels at zero deflection, applies preparation commands for 180 ms, and crosses the transition without resetting state. Persistent channel directions remain engaged. Complete role reversal crosses through the modeled dead zones.

Under the declared parameters, persistent directions produce no zero-transmission interval and a torque-error impulse of 0.0717 N m s. Role reversal produces 11.5 and 22.0 ms zero-transmission intervals and 0.1954 N m s error impulse. From a relaxed start both programs perform similarly poorly. Set dead-zone width to zero and the strategy advantage disappears. That negative control identifies the modeled mechanism.

14.2.4 Compliance Can Be Useful

Eliminating all compliance is not an implied goal. Tendons and shafts store and return energy. Muscle short-range stiffness can improve stability. Compliance can filter impacts and permit coordination. Excessive co-contraction costs metabolic energy and may raise joint loads. The desirable quantity is task- appropriate stiffness and preload, not maximum rigidity.

The wrist-led extreme may reduce some proximal load, use elastic storage, or aid participant-specific accuracy. The current objective does not price those benefits. Its conditional continuity result therefore cannot select a universal winner.

14.2.5 What Must Be Measured

A serious human test needs bilateral six-axis grip wrenches and pressure, closed-chain inverse dynamics constrained by those measurements, wrist and shaft kinematics, shaft strain, transition-aligned force-development delay, and EMG from forearm, shoulder, and scapular muscles. Perturbation or system-identification methods should estimate tangent stiffness. Participant holdout prevents a group average from becoming a prescription.

Model Result: Persistent load direction outperforms complete role reversal only within the declared dead-zone transmission family; the advantage vanishes when that mechanism is removed.

Human Evidence: Scapular and wrist EMG show phase-dependent activity but do not identify hand-force direction or net club-moment allocation (Jobe et al., 1995; Robinson et al., 2023).

Hypothesis: Proximal push-pull during preparation may preserve useful transmission continuity while wrists resist, but the opposite allocation could win on load, accuracy, or participant-specific objectives.

Practical Interpretation: “No slack” should become a measurement question: was there a force gap, low tangent stiffness, delayed transmission, or merely a different internal allocation?

14.2.6 Two Coherent Extremes

The first extreme begins with arm and shoulder-girdle action supplying a push– pull tendency at the handle during the backswing. The wrists resist enough to maintain the desired club relationship. Around transition, the arm-level drive keeps its broad direction while the coupled system reverses motion. In a model with a direction-dependent transmission state, this continuity can avoid crossing an unloaded zone before force rises again.

The second extreme begins with wrists supplying more of the club-turning moment while arm action provides resistance or support. In the downswing, arm action becomes the stronger positive driver and wrists oppose to retain the club. This requires a more explicit role reversal at the actuator level. It may still be effective if activation, elasticity, and contact remain loaded through the transition rather than becoming mechanically disengaged.

Neither extreme is a complete description of scapular motion or muscle action. They are generalized allocation families constructed to make contrasting predictions. A participant may use an intermediate allocation, and lead/trail roles may differ across anatomical axes.

14.2.7 Define Slack Operationally

"Slack" can mean at least four things: literal loss of contact, low force in an otherwise closed contact, a dead zone in a compliant transmission, or low effective stiffness caused by activation and tissue state. These have different measurements. Grip separation or visible motion could reveal contact loss; bilateral force reveals load; perturbation response estimates stiffness; EMG and ultrasound add information about activation and tissue behavior.

The model uses a declared phenomenological dead-zone transmission, not literal anatomical slack. Its state records whether force transmission remains engaged when command direction changes. The parameter can be set to zero as a negative control. If the continuous-direction advantage vanishes, the result is conditional on that mechanism. If it persists, another pathway must explain it.

A newer audit keeps five candidate mechanisms separate: contact disengagement, transmission dead zone, structural preload, biological series compliance, and control deadband. All five can produce distinguishable local parameter sensitivities in a synthetic scalar test, yet contact and biological surrogates remain nearly indistinguishable from one transmitted-output channel. This is a useful negative result: a clean-looking force trace cannot by itself tell a reviewer which kind of "slack" was present. Forward delivery and human tests therefore need independent contact, shaft, tendon, activation, and bilateral wrench measurements.

14.2.8 Why Continuous Loading Might Help

Force does not appear instantaneously in biological tissue or actuators. Electromechanical delay, tendon compliance, muscle short-range stiffness, grip deformation, and neural feedback all introduce history. Maintaining some load through transition might reduce the time required to develop a useful wrench in the early downswing and could improve resistance to perturbation.

But continuous load also has costs. It can increase co-contraction and internal joint force, reduce movement freedom, create fatigue, or interfere with a useful reorientation. "No slack" is therefore not automatically optimal. The correct question is whether a measurable loaded state improves a declared outcome at an acceptable internal cost compared with alternatives.

14.2.9 Why Role Reversal Might Help

Reversal can exploit different muscle groups and mechanical advantages across phases. The backswing has more time and different geometry than the late downswing. A wrist-dominant backswing allocation might position or load the club efficiently, while stronger proximal structures dominate downswing work and the wrists regulate release. Antagonistic preactivation could also preserve stiffness despite a change in net torque sign.

Therefore the opposite strategy is not equivalent to "letting the system go slack." Net moment can reverse while individual muscles or hand contacts remain loaded. The experiment must measure force magnitude, differential modes, and stiffness rather than inferring engagement from net sign.

14.2.10 A Matched-Task Comparison

Construct two control allocations that produce the same net club wrench during selected backswing and transition windows. In Allocation A, generalized arm drivers carry more of the task and the wrist subspace resists. In Allocation B, the weighting is reversed. Constrain both to the same club path or terminal state, then compare peak joint demands, hand common/differential forces, activation-state changes, work, and sensitivity.

Next allow the downswing to evolve freely from matched visible geometry but keep their internal transmission states. This asks whether history matters. Finally match the internal state too. If the performance difference disappears only then, the modeled preload state mediates the effect. Such staged matching is far more informative than comparing two unconstrained swings.

14.2.11 Predictions That Separate the Extremes

The continuous arm-dominant hypothesis predicts less force-rise delay after transition, fewer or shorter near-zero-load intervals in the relevant bilateral mode, and a particular sign continuity in arm-level control. The role-reversal hypothesis predicts a clearer change in net allocation but may retain high co-contraction or contact load. Both can predict similar club motion.

If bilateral wrenches and stiffness show no systematic difference despite distinct instructions or modeled controls, the slack explanation loses support. If one allocation improves speed only by increasing peak load or face/path variance, its practical value becomes conditional. If participant preferences split, the product should report subgroups or individual response rather than a population slogan.

14.2.12 Safety and Coaching Boundary

This comparison is not an instruction to apply maximal opposing force or to manipulate the wrists aggressively. The model does not estimate tissue safety, and the linked EMG evidence does not identify a preferred strategy. Human experiments require bounded tasks, clinical and ethics review where appropriate, and monitoring of discomfort and load.

For a reader or coach, the useful principle is narrower: distinguish continuous mechanical engagement from continuous net torque, and distinguish a felt cue from the physical variable it is supposed to change. A cue becomes scientifically useful only when it reliably changes the measured mediator and outcome.

14.2.13 What Would Count Against Preload

The hypothesis is weakened if the modeled advantage remains after the dead zone and activation delay are removed; if measured transition load or stiffness does not mediate the outcome; if continuous loading raises internal cost without a robust task benefit; or if held-out participants show no repeatable relation. These conditions prevent “connection” from becoming an explanation that fits every result.

14.2.14 Reader Lab: Make the Opposite Case Strong

Begin by arguing for continuous arm-dominant loading. It can preserve a loaded transmission state, reduce a force-rise gap, and let wrists regulate rather than create the main wrench. Then construct the strongest role-reversal account: wrist action may use backswing geometry efficiently, antagonistic activation can preserve stiffness through net reversal, and stronger proximal structures can take over downswing work.

Write predictions that differ. Continuous loading should reduce a defined gap in a particular force mode. Role reversal may show a sign change in allocation without reduced stiffness. Both could yield equal club motion. Specify how bilateral wrench, perturbation response, EMG, and shaft state distinguish them.

Now test a middle family rather than only endpoints. Map allocation continuously from arm-dominant to wrist-dominant and allow phase-specific mixture. Plot speed, face/path, load, robustness, and transmission measures. If an interior region dominates both extremes, the spectrum has done its job by revealing that the original dichotomy was too coarse.

14.2.15 Interpreting a Null Result

If instructions do not change measured allocation, the manipulation failed; it does not show strategies are equivalent. If allocation changes but preload measures do not, the dead-zone explanation weakens. If preload changes without outcome, it may be mechanically real but practically unimportant. If an outcome changes without the predicted mediator, search for a competing pathway.

This staged interpretation prevents every outcome from supporting the same story. It also respects participant differences: one subgroup may maintain stiffness during reversal while another develops a measurable gap.

14.3 Where the Picture Breaks

The gearbox picture is intentionally harsh. Muscle-tendon systems have continuous force-length and history behavior, not rigid tooth gaps. Surface EMG cannot measure force or stiffness directly. Scapular retraction does not map uniquely to one hand force.

The model also treats two abstract channels, not dozens of muscles and joint axes. Its numerical advantage is a conditional prediction awaiting data, not a coaching instruction.

Go Deeper: Read the full arm-wrist allocation and preload chapter, the completed epic record, and the linked scapular and wrist activation studies above.

Chapter 15

The Shaft Is a Spring With a Memory

15.1 A Concrete Picture

Bend a flexible ruler and release it. The ruler does not respond only to the force you apply now. Its present shape and motion carry a short history of earlier loading. It can return energy after your hand force has fallen, overshoot, and vibrate in several patterns.

A golf shaft behaves far more subtly than the ruler, but the central point survives: flexibility adds state. Treating the club as perfectly rigid may be adequate for one question and misleading for another, especially when late timing, grip moment, and impact-window energy are under study.

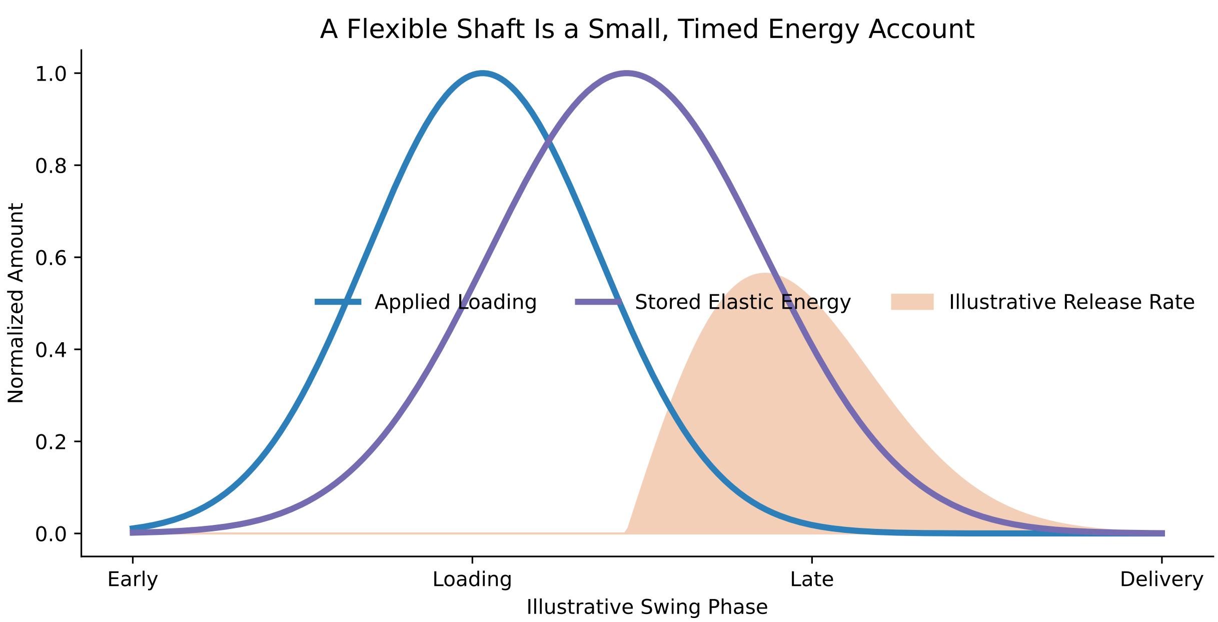

Figure 15.1: A Flexible Shaft Accepts Power, Stores Elastic Energy, and Returns Part of It Later While Damping Removes Some.

15.2 How the Mechanism Works

The simplest flexible model adds one coordinate s for shaft deflection. Its stored elastic energy might be $U_s = \frac{1}{2}ks^2$, and damping force might be proportional to $\dot{s}$. The state now includes s and $\dot{s}$. Grip work can increase U_s; later shaft motion can return energy to rigid-body coordinates.

15.2.1 Timing, Not Free Speed

A spring conserves energy only in an ideal undamped model. It cannot create net energy. Its value is timing: accepting work when the input is available and returning it when geometry or task makes that energy useful. Damping and impact can prevent full return.

In the matched rigid-flexible reference, flexibility changes delivery timing modestly rather than producing a magical speed reservoir. The work-energy ledger tracks grip input, gravity, damping, stored energy, and rigid kinetic energy so any benefit has a source.

15.2.2 One Mode Versus a Distributed Shaft

A real shaft bends continuously. Modal analysis represents its shape as a sum of vibration patterns. The first mode may capture most low-frequency deformation; higher modes matter when excitation is rapid or spatially complex. A one-mode model is efficient but should be compared with a higher-order beam reference.

The technical study assembles an Euler-Bernoulli beam with tip mass and rotary inertia, computes modal frequencies, identifies synthetic stiffness and mass, and compares reduced responses. It then couples several modes into forward two-hand dynamics. Mode truncation is judged by common wrench, power, energy, and trajectory errors, not by appearance alone.

15.2.3 Excitation Decides Required Complexity

Slow loading may barely excite high modes, making a rigid or one-mode model adequate. A sharp command, impact, or rapid contact change can excite them. The right model therefore depends on the frequency content of the question.

This is analogous to describing ocean swell with one broad wave versus describing a splash. More modes are not inherently more truthful; they are needed when the input and observable are sensitive to them.

15.2.4 Interface Power Connects Hand and Shaft

At the grip, the equivalent force and moment act on a moving, rotating, deforming interface. Consistent power combines translational force power, rotational moment power, and modal generalized force power. If those terms are omitted or double counted, stored energy appears from nowhere.

The flexible model separates grip moment from internal shaft moment. A negative grip couple can coexist with shaft energy release and positive club kinetic- energy growth. This makes the shaft relevant to the negative-torque discussion without turning it into the sole explanation.

15.2.5 Impact Is a Severe Boundary

Near ball contact, high-frequency response and contact duration matter. A model that stops at a pre-impact criterion can discuss delivery but not claim a full impact prediction. A distributed shaft needs measured stiffness, damping, geometry, clubhead inertia, grip conditions, and timestep resolution before it becomes equipment calibration.

Model Result: Reduced and distributed shaft tiers pass declared modal, work-energy, and timestep checks for synthetic reference parameters. They show conditional timing and coupling effects.

Human Evidence: Shaft deformation can be measured with strain or high-speed motion, but the current release does not identify a participant's shaft or grip properties. Measured stiffness effects also depend on club presentation and wrist kinematics (Betzler et al. 2012).

Hypothesis: Shaft state may mediate how a late negative grip couple and proximal power coexist, returning stored energy while the grip channel opposes rotation.

Practical Interpretation: Flex behaves like short-term mechanical memory: what the shaft does now depends on how it was loaded and how fast that loading changed.

15.2.6 What the Shaft Adds to the State

A rigid club can be described by position and velocity. A flexible shaft needs additional deformation coordinates and their rates. A one-mode model might use one bending amplitude; a distributed beam uses many modes or elements. These states carry memory because current shaft force depends not only on grip and clubhead position but also on how the shaft is bent and moving.

That memory complicates matched-state comparisons. Two swings with identical grip and clubhead pose can have different shaft curvature or modal velocity. Removing a torque from those apparently identical frames can produce different responses. A serious flexible-club study therefore records and matches the complete shaft state, not just rigid coordinates.

15.2.7 Loading, Storage, and Return

When grip and clubhead motion bend or twist the shaft, hand and inertial forces do work that increases strain energy. Damping turns some mechanical energy into heat. As deformation changes, the shaft can return energy to the rigid motion of grip and head. Return is not guaranteed to occur in the useful direction or at the useful time; phase matters.

It is tempting to say the shaft "kicks" the clubhead. A better account plots strain energy, damping loss, power at grip and head interfaces, and modal phase. If shaft energy decreases while clubhead kinetic energy increases, return may be part of the explanation, but whole-system closure and other power ports still matter.

15.2.8 A Simple Spring Thought Experiment

Take a spring with stiffness k and displacement x. Stored energy is $\frac{1}{2}kx^2$. Doubling deformation quadruples energy, but a real shaft's distributed bending and torsion do not reduce exactly to one scalar spring. Add damping and a changing boundary, and release timing depends on natural frequencies, mode shapes, and excitation history.

If the downswing duration aligns poorly with a mode, extra bending can return too late or increase presentation variability. A stiffer shaft stores less for the same load but responds faster; a more flexible shaft stores more but changes geometry and phase. There is no universal “more flex equals more speed” rule.

15.2.9 Interaction With the Negative-Couple Mechanism

A two-hand couple loads the handle and can excite shaft bending or torsion. Shaft reaction then changes the wrench at the hands and the clubhead’s future state. A negative contact couple may partly regulate deformation, or it may be a reaction to deformation generated elsewhere. Causal separation requires killswitches and energy accounting in the flexible model.

Compare rigid, one-mode, and distributed-shaft rungs under equivalent initial conditions. Does the late couple retain sign? Does its timing shift with modal frequency? Does a speed benefit remain after shaft strain energy and damping are accounted for? If a mechanism vanishes in the flexible rung, that is important evidence about the rigid approximation.

15.2.10 Interaction With Preload and Slack

Continuous hand loading can maintain shaft deformation through transition, while a role reversal can unload and reload one mode. Yet “shaft preload” is not the same as muscle or grip preload. The shaft may remain bent while a hand-force component crosses zero, or vice versa. Each state requires its own measurement.

The phenomenological dead zone should not absorb shaft dynamics implicitly. If shaft deformation is modeled, transmission parameters and modal states must be identifiable enough to avoid two mechanisms explaining the same delay. Ablation tests—rigid shaft, zero dead zone, altered damping—help show which mechanism actually produces the result.

15.2.11 Model Validation Ladder

Begin with static deflection under known loads. Continue with free-vibration frequency and damping tests, then dynamic boundary excitation. Verify modal convergence and timestep stability. Compare strain, tip displacement, and energy balance with an independent beam or finite-element reference. Only then embed the shaft in the full swing.

Human validation uses instrumented clubs with calibrated strain or optical shape measures. Club properties should be measured, not selected merely from a label such as “stiff.” Repeated trials reveal whether deformation phase and contact wrench are consistent within a golfer. Equipment changes should be bounded and randomized to reduce learning and expectation effects.

15.2.12 What a Shaft Result Can Support

A validated model can say that, under declared properties and motion, certain modal forces, powers, and energy exchanges occurred. It can predict how a controlled property change shifts them. It cannot infer a golfer’s preferred feel, injury risk, or universal fitting recommendation without additional data.

For the proximal-distal framework, the shaft is most valuable as a test of mechanism survival. If the interaction and allocation conclusions remain under realistic flexibility and uncertainty, confidence grows. If not, the model ladder has found the boundary where the simpler explanation must be revised.

15.2.13 Reader Lab: Watch Phase Change the Answer

Excite a calibrated one-mode shaft with the same peak handle force at three different times. Plot deformation, modal velocity, strain energy, grip power, head power, and damping loss. The same force magnitude can store or return different energy because phase and boundary motion differ.

Copy two states with identical rigid club pose but opposite modal velocity. Apply the same future hand wrench. Clubhead response should differ. Then set shaft deformation and modal velocity to zero while retaining rigid state; this ablation measures what shaft memory contributes under the model. It is an artificial counterfactual, not an available human action.

Repeat the negative-couple branch across stiffness and damping. Report whether couple sign, onset, work, and terminal speed remain. A result that survives realistic property uncertainty deserves more confidence than one tuned to a single resonance.

15.2.14 Avoiding Double Explanations

If both a dead-zone transmission and flexible shaft can produce delay, fit or vary them jointly. Look for observables that separate them: grip force may change before shaft strain under one mechanism, while modal phase dominates another. Without such information, their parameters can compensate and neither is identifiable.

The release should therefore include ablations and joint sensitivity, not one best-fit flexible trace. When two explanations remain equivalent, the honest output is an experiment recommendation—usually synchronized wrench and shaft state—rather than a preferred story.

15.2.15 A Minimum Flexible-Club Result

At minimum, publish the measured or declared club properties, modal frequencies and damping, deformation and modal velocity, grip and head power, strain-energy history, damping loss, and work-energy residual. Show timestep and modal convergence. Compare rigid and flexible branches from matched state.

Then qualify the conclusion. A synthetic property sweep shows mechanical possibility and sensitivity. A calibrated participant club supports prediction for that equipment. A replicated held-out result supports a broader statement. Skipping these levels turns shaft "memory" into a metaphor too broad to fail.

15.2.16 The Registered Test Says the Benefit Is Not Universal

That ladder has now been climbed one full rung, and the answer is worth stating plainly because it contradicts the comfortable version of the story.

A passive shaft — two first bending modes plus tip twist, bending anchored to a 24-element beam reference at 5.2399 Hz and declared torsion at 70.1260 Hz — was added to the articulated model and run for 384 trajectories against a rigid comparator from matched states. Deflections stayed small, as the linear-domain screen requires: at most 1.696 mm of tip bend and 0.00186 rad of twist. Two coarse-timestep probes that left the linear domain were excluded and reported rather than quietly dropped.

Then the honest comparison. Only **126 of 384** coupled-versus-rigid cells match within 5% on both peak load and dissipated work — comparisons across unmatched cells are not like-for-like and are set aside. Among the matched cells, the final club-speed difference runs from -0.0285 **to** $+0.0212$ **m/s**. Both signs appear. Sometimes the flexible shaft is slower.

So "the shaft gives you speed" is **rejected as a universal rule** at this tier. Not disproved for every club, golfer, or delivery — rejected as something you can assume. Whether a particular shaft helps a particular swing is a question that still needs matched states, a closed energy ledger, and calibrated equipment.

Two limits keep this result in its lane. The structural numbers are synthetic references, not a calibrated club: a committed six-mode beam disagrees materially more under short high-frequency loading than under slow loading, so short sharp events are exactly where one mode is least trustworthy. And ground reaction and free moment are absent from this tier, so nothing here speaks to what the feet contribute.

15.2.17 Chapter Checkpoint

Before crediting the shaft with a speed effect, verify that deformation was represented or measured, the modal state was included in comparisons, shaft energy decreased consistently with return, damping was accounted for, and the whole ledger closed. Then test whether the outcome survives property uncertainty and an independent flexible model. Note that when this was actually done, the speed benefit did not survive as a general rule.

The remaining biological question is individual: whether a golfer and club repeatedly create the predicted phase relation without unacceptable control or load cost. That requires calibrated human data.

15.3 Where the Picture Breaks

The ruler bends in one obvious plane. Shafts bend and twist in space, taper, vary in material, and interact with a distributed grip and clubhead. Euler-Bernoulli assumptions omit shear deformation and some rotary effects. Synthetic parameter recovery proves a pipeline, not equipment validity.

The phrase "kick" can also imply a free late boost. Any returned energy was previously supplied and survived damping. A fair comparison counts the complete cycle.

Go Deeper: Read the gravity, damping, and shaft contribution chapter, the beam reference, and the forward modal coupling.

Chapter 16

The Ground Is Part of the Conversation

16.1 A Concrete Picture

Jump from a small boat and the boat slides backward. Push against a wall and the wall returns a force through your body. Walking, throwing, and swinging all rely on external contact with the environment. The ground is not merely a stage; it is one boundary through which force and sometimes power enter the body.

Force plates make this interaction visible, but their curves are easy to overinterpret. A vertical-force peak does not identify which muscle created it. A center-of-pressure shift does not prove a unique segment sequence. The ground reaction is the net contact required by the whole measured motion and dynamics.

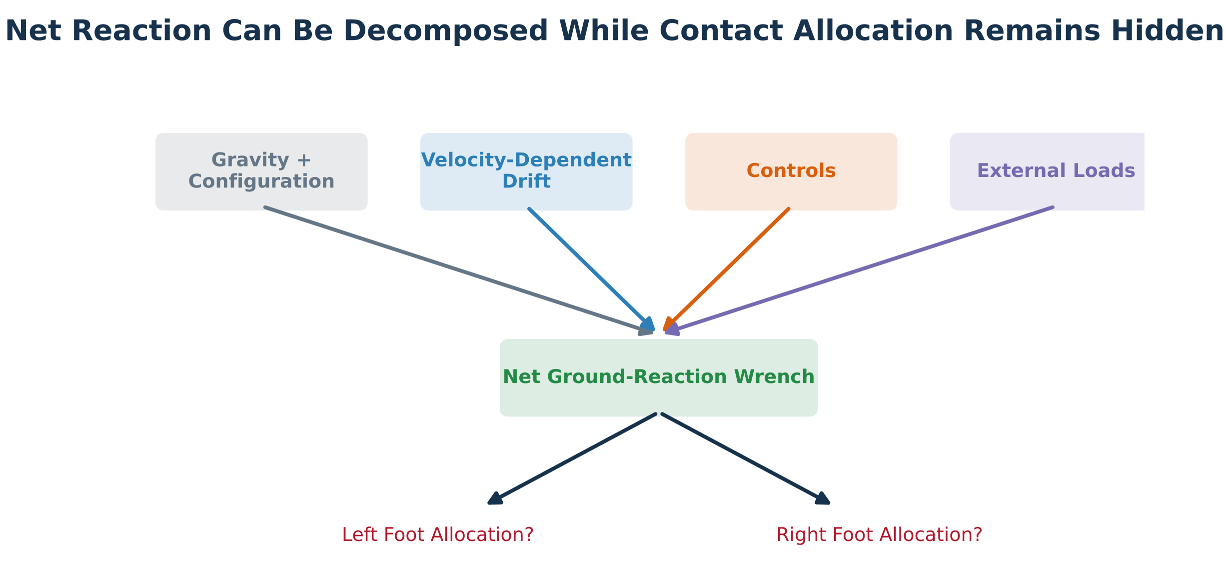

Figure 16.1: Ground Reaction Is Decomposed Into State-Dependent, Control-Dependent, and Unresolved Contact-Allocation Contributions.

16.2 How the Mechanism Works

In constrained dynamics, ground reactions appear as multipliers enforcing foot contact. Solving the equations can decompose reaction at a fixed state into configuration/gravity, velocity-dependent, control, and external-load terms. The technical study defines ground-reaction versions of zero-torque and zero- velocity counterfactual forces.

16.2.1 The Reaction Reflects the Whole System

Accelerating the club changes body reactions. Body motion changes required foot forces. Gravity contributes. If the feet are constrained, the solver finds a reaction satisfying both motion and contact. A large ground force is not a separate motor delivering energy directly to the club; it is part of the coupled external interaction.

Ground power depends on contact-point velocity. An ideal stationary foot contact has zero mechanical power despite large force. Real feet deform, center of pressure moves, and segments rotate, so joint and soft-tissue work still occurs. The force plate measures wrench, not metabolic source.

16.2.2 ZTCF Family and ZVCF Ask Narrow Questions

At one fixed state, a zero-torque counterfactual sets selected controls to zero and recomputes the reaction. A zero-velocity counterfactual sets velocity terms to zero while holding configuration. Differences reveal modeled contributions to reaction. They do not describe a human who instantaneously stopped all motion or muscle torque.

In a fixed-support double-pendulum benchmark, pointwise ZTCF predicts the general reaction waveform but retains substantial amplitude error. Opposing control can make one ZTCF impulse exceed the total because contributions cancel. This is a useful warning against treating a component as a measured fraction.

16.2.3 Two Feet Add Allocation Ambiguity

A single net wrench under the body may be distributed across left and right feet in many ways, especially if both contact regions can support moments. Separate plates reduce ambiguity. Pressure insoles add spatial information. Yet foot models, friction, and coordinate transforms remain.

The same common/differential logic used for hands applies: net external wrench is observable; internal allocation and co-contraction require more data.

16.2.4 What Ground Data Can Test

Synchronized whole-body kinematics, separate force plates, club motion, and bilateral grip wrenches can test whether model-predicted reaction components align in timing and direction. Residual forces and moments reveal inconsistencies in segment parameters or filtering. Participant holdout tests whether a fitted pattern generalizes.

Model Result: The fixed-support benchmark establishes a reproducible reaction decomposition with partial waveform agreement and explicit amplitude limitations.

Human Evidence: Golfers exhibit phase-dependent vertical, shear, center-of- pressure, and free-moment patterns, but these net observations do not uniquely identify the proximal-distal mechanism (Han et al. 2019; Ball and Best 2007; Watson et al. 2026).

Hypothesis: State-dependent reaction components may explain part of observed ground-force timing without requiring each peak to be directly commanded, but full-body human data must test the prediction.

Practical Interpretation: The ground tells us what net external conversation occurred at the feet. It does not provide a transcript assigning every sentence to a muscle.

16.2.5 Force Plates Measure a Net External Wrench

A force plate reports three force components and three moments at its origin. From these, with assumptions, researchers derive center of pressure and free moment. Separate plates under each foot preserve left–right allocation better than one plate under both. Pressure insoles add spatial distribution but usually with different accuracy and shear limitations.

The plate does not report muscle force, joint torque, or energy "from the ground." It reports the reaction required at the contact boundary. To understand how that reaction relates to the club, synchronize it with body and club motion and solve the dynamics with uncertainty.

16.2.6 Large Force Does Not Necessarily Mean Large Power

Power at an ideal stationary contact point is force dotted with zero velocity, so it is zero even when force is large. The body can still gain mechanical energy because muscles do work internally while the ground supplies external constraint. Real foot deformation and rolling complicate the point, but the distinction remains: force magnitude is not an energy-flow rate.

This is similar to pushing a locked wall during a squat. The wall or ground provides a reaction that permits body acceleration, while chemical energy in muscle supplies work. Saying the golfer "gets energy from the ground" can be a useful cue, but it is not a precise mechanical ledger unless contact work is actually calculated.

16.2.7 A Whole-Body Reaction Walkthrough

Early in the downswing, changes in pelvis and trunk motion require horizontal, vertical, and rotational ground reactions. As arms and club accelerate, their momentum changes alter the reaction needed by the rest of the system. Late club acceleration can therefore appear in foot-force features even when no new foot command specifically targets the club.

The influence is reciprocal. Ground reaction enables changes in whole-body momentum that shape proximal motion, and distal acceleration changes the reaction required for balance and constraint. Cross-correlation of peaks cannot determine direction of causation. A forward model and perturbations are needed.

16.2.8 Pointwise Ground-Reaction Decomposition

At a copied state, solve the constrained equations normally, then with selected torque drivers removed. The difference in constraint multiplier is the modeled control-dependent contribution to ground reaction.

Set velocities to zero in a separate calculation to isolate velocity-dependent terms. Keep gravity and configuration unchanged.

Components can cancel, so a counterfactual reaction may exceed total in one direction. That does not mean it contributed more than one hundred percent of a measured force. It means another component opposed it. Report signed waveforms, impulses, cancellation, and residual rather than forcing an additive percentage story.

16.2.9 Two Feet Create an Allocation Problem

The same net body wrench may be supported by different left and right contact wrenches within friction and balance constraints. Separate plates observe much of this allocation. If both feet occupy one plate, a model or pressure data must supply the split, and nonuniqueness should be shown.

Foot models also need contact assumptions. A fixed point, rolling contact, distributed pressure, and compliant shoe produce different reaction paths. The choice should match the measurement question. A mechanism that depends on an unrealistic fixed foot may not survive a richer contact rung.

16.2.10 Connecting Ground and Hand Ledgers

Whole-system analysis links external ground and gravity wrenches to momentum change. Segment and joint analyses trace how internal powers and reactions relate to the hand–club interface. The links must close simultaneously: body plus club energy, body momentum, hand action–reaction, and foot reaction.

This creates powerful error checks. If measured hand and ground loads combined with motion leave large residuals, segment inertias, filtering, synchronization, or unmeasured contacts may be wrong. A polished transfer story should wait until those residuals are bounded.

16.2.11 What Ground Manipulations Could Test

Researchers can compare stance width, lead/trail pressure constraints, surface friction within safe limits, or externally paced tempo while measuring the predicted hand and club mediators. The mechanism should specify whether changing a ground variable shifts proximal state, hand wrench, negative-couple interval, or only balance strategy.

A finding that ground-force peaks change without the predicted hand or club change would weaken a simple ground-to-club transfer narrative. A finding that hand-wrench timing mediates the relation would strengthen the chain. Participant- held-out analysis is essential because people may adopt different compensations.

16.2.12 Practical Boundary

Force-plate patterns can characterize an individual and expose constraints, but they should not become universal target traces. Similar club outcomes can arise from different foot allocations, and larger peaks can reflect braking or balance as readily as useful propulsion. The valuable question is how the measured external wrench fits the complete system ledger and whether a proposed change improves robust outcomes without unacceptable load.

16.2.13 Reader Lab: Close the External Wrench

Take synchronized body and club kinematics, segment parameters, separate foot wrenches, gravity, and hand/club loads. Compute whole-system linear and angular momentum change and compare with integrated external wrench. Plot residual by direction and phase. A large residual near peak club acceleration may reflect synchronization, filtering, inertia, or omitted contact—not a new force.

Next sum the two foot plates and verify the result against a single combined wrench calculation. Shift reference point consistently. Compute center of pressure only when vertical force and moment assumptions make it well defined. Flag low-force intervals rather than allowing unstable ratios.

Finally, run pointwise zero-torque and zero-velocity model decompositions at the measured state. Compare predicted component timing with observed reaction but do not call the components measured. Their mismatch becomes a validation residual and a guide to missing dynamics.

16.2.14 Competing Explanations for a Force Peak

A vertical peak can reflect accelerating the body center of mass, resisting downward motion, club reaction, balance, or a combination. A horizontal peak can turn or translate the system. A free moment can relate to rotational balance but does not map uniquely to pelvis or club torque.

Test proposed pathways with mediation and intervention. If a stance manipulation changes ground force but neither proximal state nor hand wrench as predicted, the simple transfer chain is unsupported. If all links change in order and a forward model predicts them on held-out trials, the explanation strengthens.

16.2.15 From an Observation to a Causal Claim

Suppose higher club speed is associated with a larger lead-side vertical impulse. That is an observation. A causal hypothesis might say the impulse changes pelvis and trunk state, which changes hand wrench and later club acceleration. The hypothesis requires each mediator and its timing. A model can predict the chain; a manipulation and synchronized data can test it.

Alternative explanations include stronger golfers producing both variables, different stance or club, balance response to the faster club, or processing choices. Randomized safe manipulations, within-person contrasts, and held-out prediction reduce these alternatives. The ground trace alone cannot.

16.2.16 What the Product Should Display

Show separate and summed foot wrenches, center of pressure only in valid intervals, whole-system momentum residual, predicted reaction components, and uncertainty. Link peaks to concurrent body, hand, and club state. Let readers toggle pointwise ZTCF and ZVCF components while explaining that they are model counterfactuals.

Do not rank a golfer from force magnitude alone. Normalize only when the scientific comparison justifies it and retain dimensional values. A transparent display should make cancellation, allocation ambiguity, and residuals as visible as the headline pattern.

16.2.17 A Final Ground Check

If an ideal fixed foot does zero work, positive whole-system energy must come from modeled internal actuation and other external ports, not be credited to the stationary ground. If compliant foot contact does work, define the moving contact point and material energy. This distinction keeps everyday "use the ground" language from corrupting the energy ledger.

16.2.18 What Happened When the Ground Was Actually Added

This is no longer hypothetical. A finite planar base was added to the articulated model — ground translation, a free pitch moment, and a coupled pathway — each compared against an exact fixed-base version of the same model from the same state. That is 384 primary runs plus 192 control runs, on two independent physics engines. The numerics are clean: energy residuals halve when the timestep halves, the two engines agree to ten decimal places, and no run fails a check.

The result is the sharpest negative in the whole program, and it is worth understanding rather than skipping.

The test declared **in advance** that a fair comparison requires matching both peak grip load and total dissipated work to within 5%. **Zero of 384** cells passed. Peak load actually agrees well. Dissipated work does not, and cannot: only the version with a moving base has ground damping, so it dissipates roughly twice as much energy by construction. You have to loosen the tolerance to 200% before anything matches — at which point everything matches and the test has stopped meaning anything.

So the honest summary is: **at this tier, a ground pathway benefit was not established.** Not refuted for real golfers — not established by this model.

Now the part that requires discipline. If you ignore the matching requirement and just compare final club speed at 50 ms, the moving-base runs are faster in all 48 cells, by up to 0.177 m/s. That looks like a clean win for "use the ground." It is not. Those are exactly the cells that failed the fairness check — they are comparing runs that dissipated different amounts of energy. Quoting that number as a ground benefit would invert the study's actual finding.

A separate, clearly labeled follow-up asked a narrower question: if you count only grip and shaft dissipation and set ground damping aside, 60 cells match, and their speed differences run from −0.0008 to +0.0139 m/s — 20 faster, 40 slower. That is a sensitivity check chosen after seeing the primary failure, not a replacement verdict, and mostly the sign is negative.

One more lesson came free. The same model, started three different ways, gave peak ground forces of 33, 565, and 510 N and 4 ms club speeds of 0.26, 1.91, and 0.95 m/s. How you initialize a standing model is not a technicality; it can swing the answer by an order of magnitude.

Keep the scope in view: this ground is a linear spring in a plane plus a pitch moment. It is not two feet, not friction, not pressure under the toes, and it is not calibrated against a force plate. It also stops at 50 ms.

16.2.19 Chapter Checkpoint

Ground reaction is external force evidence, not a muscle transcript or automatic energy source. Use it to close momentum, test contact models, and constrain the causal chain through body and hands.

Report left/right allocation and model components without converting them into additive percentages.

And when someone shows you a speed difference between a moving base and a fixed one, ask what was held equal before the comparison was made.

A practical pattern becomes mechanistically meaningful only when its predicted mediators and outcome survive within-person manipulation and participant holdout.

16.3 Where the Picture Breaks

The wall and boat pictures use simple rigid contacts. Feet roll, deform, slip microscopically, and distribute pressure. Fixed-support pendulums omit balance, pelvis translation, and bilateral contact.

Correlation between force peaks and segment motion cannot establish causation. Timing can reflect shared state, feedback, or filtering. Reaction decomposition must be validated against synchronized data before becoming a human claim.

Go Deeper: Read the monograph's ground-reaction drift attribution and its human falsification protocol.

Chapter 17

When the Base Moves, Every Shortcut Changes

17.1 A Concrete Picture

Stand on a stable floor and swing a pendulum from your hand. Then repeat while standing on a rolling board. The pendulum pulls you; your motion changes its path; the board moves beneath both. Prescribing your hand path from the first experiment and replaying it on the board would erase this feedback.

Many simple golf models prescribe a hub, shoulder, or hand trajectory. That is useful for asking what club forces are compatible with a measured path. It is less useful for asking how club reactions alter the body. A moving-base forward model turns the prescribed support into a dynamical participant.

17.2 How the Mechanism Works

In the simplest double pendulum, the shoulder may be fixed. In a prescribed- motion model, shoulder or hub kinematics are supplied as known functions of time. The solver computes joint reactions needed to realize them. In a coupled model, base coordinates join $\mathbf{q}$ and their accelerations emerge from force and inertia.

17.2.1 Prescribed Motion Is an Infinite-Authority Driver

A perfectly prescribed trajectory occurs regardless of reaction. Mathematically, the driver supplies whatever force and power are needed. This can hide a large back-reaction from the club. The computed wrench is still useful, but the model cannot predict how the body would deviate under finite strength or mass.

Calling prescribed motion "measured" does not remove this limitation. It makes the model an inverse or hybrid analysis of that trial. Causal strategy comparison needs a forward tier where changes in control can change the base state.

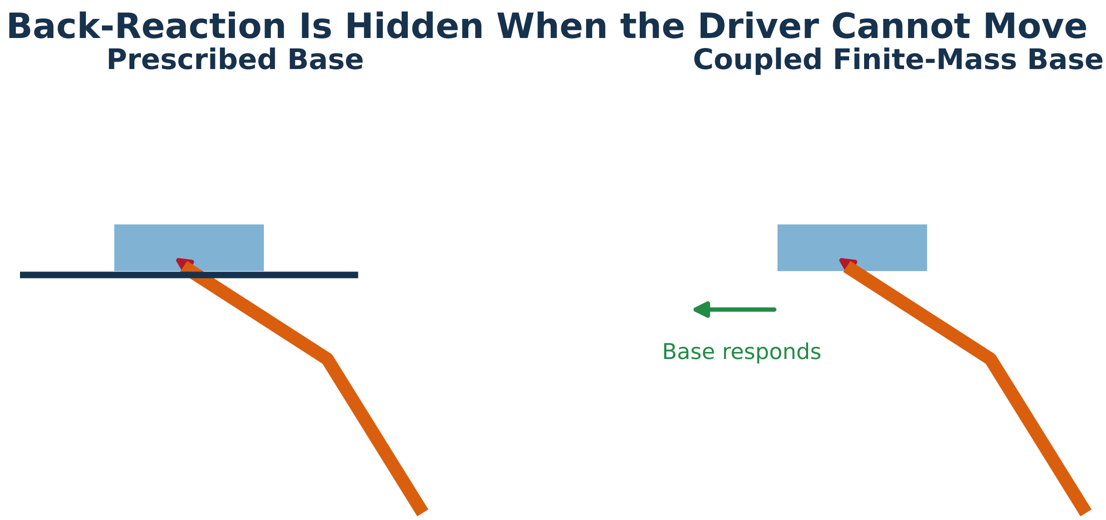

Figure 17.1: A Prescribed Base Follows a Fixed Path While a Coupled Moving Base Responds to the Club Reaction.

17.2.2 Coupled Inertia Creates Feedback

When base and club coordinates share one mass matrix, off-diagonal terms connect their accelerations. A club reaction can slow, redirect, or translate the base. Base motion changes hand velocity and therefore club power. The whole-system balance must include both.

The coupled base-flex model separates grip and shaft moments, includes convective terms, and uses a constrained solve. Same-state interventions remove selected drivers or reverse geometry while preserving the instant. Sensitivity sweeps ask whether the negative-couple mechanism survives plausible parameters.

17.2.3 Finite Mass Matters

An infinite-mass base barely responds; a light base moves strongly. Human body segments have finite, distributed mass and active control. A reduced moving hub does not become a torso, but it tests whether a mechanism depends on an immovable support.

The independent spatial test uses finite-mass hand carriages rather than direct club actuation. Paired compliant contacts transmit action-reaction. The late negative couple survives for a bounded interval in both engine realizations. This broadens the mechanism beyond one prescribed planar driver.

17.2.4 Whole-System Power Prevents Hidden Work

A prescribed base can inject or remove power implicitly. A forward base requires explicit drivers or external forces. Summing actuator work, gravity, damping, contact power, shaft storage, and kinetic-energy change exposes hidden sources. Timestep refinement distinguishes physical residual from numerical error.

17.2.5 A Hierarchy of Questions

Use fixed-base models for clean local coupling. Use prescribed motion to attribute forces along a measured path. Use a moving reduced base to test back-reaction. Use full-body models to represent spatial mass distribution and joint structure. Each tier should inherit common observables so the mechanism can be compared.

Model Result: The negative-couple and transfer findings survive a finite-base, forward constrained tier under registered controls, while quantitative values change.

Human Evidence: Measured hub paths and the modeled effect of inward hand-path motion show why base and handle translation cannot be treated as decorative details (Nesbit and McGinnis 2009; Miura 2001).

Hypothesis: Human proximal control may arrange a moving base whose reaction with the club supports late transfer, but the reduced hub cannot identify the specific body coordination.

Practical Interpretation: If the club can push back on the golfer, a model that forces the hands along an unchangeable path answers only part of the story.

17.2.6 Why a Fixed Pivot Is Both Useful and Dangerous

Fixing the proximal joint makes the double pendulum easy to interpret. It removes translational base work and exposes arm–club coupling. Yet a golfer's hands travel because pelvis, trunk, shoulder girdle, arms, and wrists all move. The hand path has curvature, inward motion, elevation change, and out-of-plane components. Once those are important to the question, a fixed pivot can distort both force direction and energy flow.

The danger is not that every fixed-base result is false. It is that a mechanism may rely on compensation for an omitted base motion. The correct response is a model ladder: preserve the same observable definitions and controls while releasing the base in stages.

17.2.7 Prescribed Motion Versus Dynamic Motion

A prescribed base follows a measured or designed path regardless of reaction from the club. This is computationally convenient and can test how hand-path geometry influences club motion. Work needed to enforce the path is an external input and must be included in the energy ledger. The club cannot feed back to change the prescribed body trajectory.

A dynamic base has mass and equations of motion. Applied forces and torques, ground constraints, and club reactions determine its acceleration. Energy and momentum exchange are reciprocal. This is more physical but introduces control, contact, and parameter questions. Comparing prescribed and dynamic versions reveals which findings depend on one-way forcing.

17.2.8 The Moving-Handle Contribution

Clubhead velocity equals handle velocity plus the contribution of club rotation and deformation. Accelerating the handle can raise clubhead speed even before relative release. Pulling the handle inward can change angular momentum and effective geometry, a mechanism explored in golf models as parametric acceleration. Whether it helps depends on timing and the rest of the state.

Handle power can be calculated from the grip wrench and handle twist. A moving base adds translational power channels that a fixed hinge cannot represent. The negative-couple question must therefore be revisited: a locally negative rotational channel can coexist with substantial positive translational hand power.

17.2.9 Releasing the Base in Rungs

Rung one prescribes planar pivot translation. Rung two adds a finite-mass planar base driven by declared controls. Rung three adds pelvis and trunk rotation with ground constraints. Rung four introduces three-dimensional shoulder-girdle and arm motion. At each rung, calculate the same hand wrench, swing-normal couple, club power, work, and terminal outcomes.

The sign of a physical wrench projection should transform consistently. Exact magnitude and preferred onset can change. If the mechanism reverses under a small plausible base motion, the fixed-base conclusion is fragile. If it persists across a broad ensemble, it becomes a stronger candidate for human testing.

17.2.10 A Counterfactual Base Test

Copy one state from a dynamic simulation. In one branch retain proximal driver; in another remove it while retaining distal control. Observe immediate changes in base reaction, handle acceleration, club constraint force, and club angular acceleration. Then integrate both branches briefly. This traces a causal path from proximal input through moving base to distal response within the model.

Repeat with a prescribed base using the baseline path. If the club effect remains despite removal of dynamic feedback, it is mostly kinematic forcing. If it changes, reciprocity matters. Neither result is "more true" automatically; the comparison identifies the assumption carrying the conclusion.

17.2.11 Moving Bases and Timing

A clock-based wrist command can meet different geometry when base motion changes. A state trigger based on arm–club angle, handle radial velocity, or a coupling term may be more portable. But a trigger chosen after inspecting results can overfit. Candidate triggers and outcome windows should be declared before held- out evaluation.

Base motion also changes the location and orientation of the late impact zone. Alignment by impact alone can hide meaningful differences earlier. Reports should show both absolute time and state-space trajectories, with uncertainty in event detection.

17.2.12 Human Measurements Required

Whole-body motion should include pelvis, trunk, scapular/shoulder proxies, arms, hands, and club. Separate force plates and instrumented grips constrain external and interface wrenches. Segment inertias and joint centers carry uncertainty. High-speed club tracking and shaft strain add the distal state.

Soft-tissue artifact is especially important at the shoulder girdle. A moving- base model should not claim anatomical precision beyond its measurement. It can use a generalized handle driver for mechanism testing while explicitly reserving muscle and scapular interpretation.

17.2.13 Practical Meaning

The useful lesson is not a command to pull the handle along a particular path. It is that club release cannot be optimized independently of how the hand base moves. Geometry, speed, and force at the handle jointly determine club response. A strategy should be evaluated across realistic base variations and control objectives rather than on one fixed-pivot animation.

17.2.14 Reader Lab: Prescribe, Then Release

Record or design one handle path and drive a club model with it as a prescribed boundary. Calculate the external work needed to enforce that motion. Then build a finite-mass base driven to approximate the path. Compare club trajectory, handle wrench, body reaction, and energy. Differences reveal the effect of dynamic reciprocity.

Run an inward-handle-path perturbation at matched state. Hold distal torque fixed and observe radial/tangential reaction, relative release, and clubhead speed. Repeat with changed club inertia. If the outcome reverses, the proposed path effect is parameter-dependent and should be mapped, not prescribed.

Finally compare a clock-scheduled distal command with a state-triggered one across varied base tempo. A state policy may retain performance better, or it may overfit an unobservable model variable. Report both the performance and whether the trigger has a measurable proxy.

17.2.15 Common Modeling Trap

Prescribing a measured body path while also assigning body torques can double- count control: the path is already enforced by whatever unreported force is needed. Decide whether motion is input or outcome. If prescribed, report its constraint wrench and work. If dynamic, show how controls and ground contact produce it.

This distinction is crucial when attributing energy to proximal drivers. A prescribed path can inject exactly the energy required and make distal transfer look inevitable. A dynamic base exposes the energetic cost and feedback.

17.2.16 A Minimal Moving-Base Benchmark

Use one finite-mass base translating in a plane, one arm link, and one club link. Publish mass and inertia, applied base force, joint torques, initial state, and constraints. Verify momentum and energy. Compare with a prescribed trajectory that matches baseline base motion. This small benchmark can be implemented in multiple engines and isolates reciprocity before a whole human model is added.

Run the two-hand geometry controls on both versions. The physical contact-couple sign should respond consistently even if reaction magnitudes differ. Run proximal killswitches and show how base, hand, and club variables diverge. These tests create a clear bridge from the fixed double pendulum to spatial whole-body dynamics.

17.2.17 Why This Matters for Strategy

An arm or scapular action changes both force and the moving base from which the club is carried. A wrist-focused model that prescribes hand path may hide the work needed to create that path; an arm-focused model may underrepresent distal regulation. A balanced comparison includes both controls and their reaction on the base.

The useful strategy question becomes: which allocation produces a favorable handle state and club interaction across realistic body variability, with acceptable load and control? The moving-base rung makes that question computable without pretending it is already answered for a golfer.

17.2.18 Chapter Checkpoint

Ask whether the base motion is prescribed or dynamically produced, where its work enters, and whether club reaction can feed back. Compare both formulations on shared observables. A mechanism that survives this comparison has moved beyond the most consequential fixed-pivot shortcut.

17.3 Where the Picture Breaks

The rolling board has one translation. A human base has pelvis and trunk motion, joint limits, balance, ground contact, and active feedback. A finite carriage is a falsification bridge, not anatomy.

Forward models also require control choices. A wrong controller can make a good mechanism look poor. Comparisons must separate mechanism failure from controller failure and report both.

Go Deeper: See the coupled base and compliance chapter and the independent spatial contact test.

Chapter 18

How a Constrained Forward Model Solves the Next Instant

18.1 A Concrete Picture

A navigation app knows where a car is, how fast it moves, road rules, engine commands, and map geometry. To predict the next second, it cannot choose position directly. It computes how forces and constraints change acceleration, then advances velocity and position. If the predicted car drifts through a barrier, the model violated a constraint or took an unreliable step.

A forward swing model performs a more demanding version of that calculation. At every tiny step it balances inertia, gravity, velocity effects, controls, contact reactions, and flexible states, then checks that joints and contacts remain closed.

18.2 How the Mechanism Works

The mass matrix $\mathbf{M}(\mathbf{q})$ says how strongly each generalized coordinate resists acceleration and how coordinates are coupled. The vector $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ collects gravity and velocity-dependent terms. Controls and external loads form generalized force $\mathbf{Q}$. Constraints add reaction $\mathbf{J}^{\top}\lambda$.

The KKT system solves $\ddot{\mathbf{q}}$ and λ together. This is crucial: reaction force is whatever enforces the declared acceleration-level closure, not an independent input selected for a preferred result.

18.2.1 Integration Turns Local Rules Into a Trajectory

Acceleration updates velocity; velocity updates configuration. Numerical methods approximate this continuous evolution with finite steps. A step too large can miss an impact, distort a fast shaft mode, or inject energy. A stable-looking animation is not numerical validation.

The study repeats key cases at smaller timesteps. Physical results should converge while work-energy and constraint residuals decrease. Event detection locates impact consistently rather than accepting the nearest coarse sample.

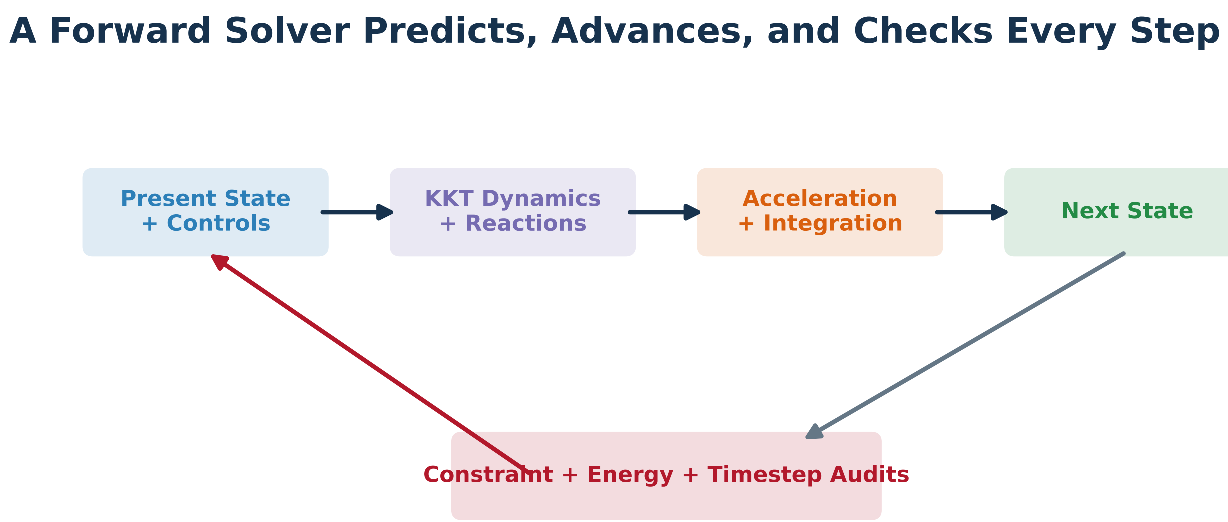

Figure 18.1: A Forward-Dynamics Pipeline Turns State and Control Into Accelerations and Reactions, Integrates the State, and Audits Constraints and Energy.

18.2.2 Constraint Stabilization Is Not Free

Roundoff and integration error can let closed contacts drift apart. Projection or stabilization corrects the state toward the constraint manifold. Corrections must be reported because large projections can hide an unstable solver. The forward tests set tolerances for position closure, velocity closure, and maximum projection.

18.2.3 Inputs Need Dynamics Too

An instantaneous torque step is mathematically convenient and physiologically harsh. First-order command filters, torque-rate limits, and activation states make inputs finite. Sensitivity to rise time tests whether a result depends on a single discontinuous sample.

The preload study goes further by evolving internal transmission state before the transition. The uncertainty study includes delayed actuators. These additions do not create a complete muscle model; they challenge timing claims with more realistic state continuity.

18.2.4 Registered Observables Keep the Solver Honest

The forward two-hand model reports club trajectory, speed, hand reactions, equivalent wrench, common and differential modes, interface powers, actuator work, energy residual, and constraint closure. A result is rejected if one key contract fails even when club speed looks attractive.

Negative controls alter contact separation, moment-arm sign, driver commands, gravity, damping, or stiffness. The solver must reproduce the qualitative and quantitative limiting behavior expected from mechanics.

18.2.5 Inverse and Forward Models Complement Each Other

Inverse dynamics begins with measured motion and estimates required forces. It is strong for trial-specific attribution but vulnerable to noise and contact nonuniqueness. Forward dynamics begins with force/control models and predicts motion. It supports interventions but depends on controller and parameter quality. Agreement between them on common observables is more informative than choosing one as universally superior.

Model Result: The constrained forward tiers pass registered closure, work-energy, negative-control, and timestep tests for their declared synthetic models.

Human Evidence: Existing three-dimensional golf models show both the value and the strong assumption load of forward simulation (MacKenzie and Sprigings 2009; Balzerson et al. 2016).

Hypothesis: Interaction-force mechanisms that persist under forward integration and geometry controls are better candidates for human experiments than effects seen only in inverse replay.

Practical Interpretation: A forward model earns trust by showing its homework: equations, state continuity, small steps, closed constraints, balanced energy, and failed cases where the mechanism predicts failure.

18.2.6 The Solver's Job at Every Time Step

A constrained forward model begins with state and control. It evaluates mass, gravity, velocity-dependent, elastic, damping, external, and applied-control terms. Constraint equations say which relative motions are permitted. The solver then finds accelerations and reaction multipliers that satisfy dynamics and constraints together. An integrator advances positions, velocities, and internal states to the next instant.

This loop repeats thousands of times. Small sign, frame, or indexing errors can create plausible-looking motion while corrupting forces or energy. Contracts at each layer are therefore as important as the final animation.

18.2.7 Coordinates and Constraints

Minimal generalized coordinates avoid explicit holonomic constraint multipliers for simple joints but can make complex contacts and spatial frames harder to manage. Maximal or Cartesian coordinates describe body poses more uniformly and enforce joints with constraints. Spatial-vector formulations organize rigid-body dynamics efficiently. Independent engines using different formulations provide valuable cross-checks.

No formulation is inherently the truth. They should agree on frame-invariant observables when representing the same physical model. Differences point to implementation, tolerance, or specification mismatch that must be reconciled before interpretation.

18.2.8 Initial Conditions Must Satisfy the Model

Positions should satisfy joint and contact constraints. Velocities should lie in the constraint tangent space. Flexible modes and controller states need declared initial values. Starting from an inconsistent state can create large artificial reaction impulses that resemble interesting interaction forces.

A projection step can place measured or designed states on the constraint manifold, but projection changes them. Record the correction and reject cases whose adjustment exceeds tolerance. For human data, measurement uncertainty should be reconciled with dynamics rather than silently forced into perfect constraints.

18.2.9 Work-Energy and Momentum Audits

The change in total modeled mechanical energy should match integrated external and control power minus declared dissipation. Constraint work should follow the contact model: zero for ideal stationary constraints, nonzero for moving or compliant contacts. Linear and angular momentum balances provide separate checks when external wrench is known.

Audits should run on every major model rung and counterfactual. A branch that removes torque changes power, but it should not suddenly create an unexplained energy residual. Report maximum constraint violation, balance residuals, integration tolerances, and refinement convergence with each result.

18.2.10 Building a Killswitch Correctly

At intervention time, copy the complete state. Modify only selected future input. Do not reset velocities, controller integrators, or shaft modes unless those are the explicit intervention. Use the same solver settings and event handling. Store both trajectories with a branch identifier and intervention manifest.

For pointwise decomposition, do not advance state. Re-evaluate the right-hand side under baseline and counterfactual inputs at the same state. This difference should close against a declared term decomposition. Keeping the two procedures separate prevents a forward divergence from being mislabeled as instantaneous force attribution.

18.2.11 Contact and Friction

Ideal bilateral constraints can transmit whatever reaction is needed. Human hands and feet have finite friction, compliance, and moment capacity. A richer model checks whether reconstructed loads remain within plausible contact cones and pressure limits. If not, the assumed no-slip constraint is infeasible.

Complementarity or compliant contact introduces switches and numerical stiffness. Event detection, regularization, and timestep can affect results. The negative-couple mechanism should be tested across reasonable contact representations rather than tuned to one solver's smoothing.

18.2.12 Verification Before Validation

Verification asks whether code solves the equations intended. Use analytic limits, manufactured solutions, finite-difference derivative checks, timestep studies, conservation cases, and independent engine comparison. Validation asks whether the intended equations represent measured reality. It uses calibrated data, uncertainty, held-out predictions, and residual structure.

A model can be verified and invalid. It can also fit data while being implemented incorrectly through compensating errors. Publication must show both tracks and avoid treating visual resemblance as either.

18.2.13 Reproducible Output Contracts

Every run should record code commit, model version, parameter manifest, control schedule, initial state, solver and tolerance, random seed, and output schema. Derived figures should be reproducible from machine-readable results. Units, frames, and sign conventions belong in the schema, not only the prose.

The open workbench should invoke the same canonical engines used by tests rather than copying equations into a second educational implementation. That reduces drift and lets a reader move from a casual experiment to the exact scientific artifact.

18.2.14 What Forward Simulation Can Establish

It can demonstrate that a mechanism is dynamically possible under declared assumptions, quantify its effects, expose tradeoffs, and make falsifiable predictions. It cannot prove that observed golfers used the simulated controls, that unmeasured muscles produced generalized torques, or that an optimum is safe. Those boundaries make the simulator a stronger scientific resource, not a weaker sales demonstration.

18.2.15 Reader Lab: Break the Solver on Purpose

Reverse one sensor axis, introduce a small initial constraint violation, enlarge timestep, remove damping, and shift one input channel in time. Each defect should trigger a predictable gate: frame closure, projection correction, convergence, energy behavior, or synchronization. If outputs remain silently "valid," the product is not fail-closed.

Then perform an independent implementation of one benchmark. Share only the specification and input manifest, not internal derivative code. Compare state, reaction, power, and energy residual. Agreement in animation with disagreement in wrench is not enough because this project interprets forces.

Finally replay a stored run from exact commit and seed. Regenerate tables and figures and compare hashes or numeric tolerances. Reproducibility should be a test, not a claim in the methods paragraph.

18.2.16 Interpreting Solver Failure

A failed integration can indicate numerical stiffness, infeasible contact, singular geometry, or a physically extreme control. Retrying with looser tolerances until motion appears is not an acceptable default. Classify failure, retain the case in ensemble statistics, and determine whether the model or numerics needs revision.

Failure regions are scientifically valuable. They reveal where an ideal constraint demands impossible reaction, where contact should switch, or where a control policy lacks robustness. The public tool should visualize these regions rather than omitting them from the success rate.

18.2.17 Model Contracts as Scientific Sentences

"Mass matrix is symmetric positive definite in the qualified state region" is both a test and a statement about kinetic energy. "Constraint violation remains below tolerance" says simulated contacts mean what the prose claims. "Wrench power is invariant under reference-point transform" protects the energy story. "The geometry reversal changes the couple sign" encodes the proposed mechanism.

Writing such contracts before implementation gives the solver an explicit job. Tests then protect scientific meaning as code evolves. A regression that passes visual inspection but violates one contract is caught before publication.

18.2.18 Extending the Engine Safely

When adding a shaft, moving base, or spatial joint, preserve old benchmarks and add new limiting cases. Version schemas rather than silently changing column meaning. Rebuild all derived artifacts and compare qualified observables. Record intentional changes in a release note and evidence manifest.

This discipline supports multiple front ends and repositories. The canonical physics library owns equations and contracts; educational pages consume its versioned outputs. Duplication is minimized, and a casual explanation cannot quietly drift from the tested engine.

18.2.19 What a Reviewer Needs

Provide a small runnable benchmark, exact command, expected tolerances, and human-readable trace. Include failed-case examples. A reviewer should not need the full optimization stack to verify core dynamics. This modularity is one of the most important ways to move the project from an internal modeling package to an open scientific resource.

18.3 Where the Picture Breaks

The navigation picture has known roads. Human joint constraints and controls are uncertain. A numerically impeccable solver can integrate the wrong anatomy or controller. Verification checks code against equations; validation checks model predictions against reality.

Contact models are particularly consequential. Rigid, compliant, frictional, and distributed contacts can change internal forces. Results should be qualified by the contact family tested.

Go Deeper: Read the forward constrained two-hand model, the solver and platform methods, and the open qualification instructions.

Chapter 19

A Ladder of Models, Not One Final Model

19.1 A Concrete Picture

A map can show a continent, a city, a building, or the pipes inside one room. The city map is not false because it omits plumbing. The plumbing drawing is not better for planning a cross-country trip. Scale and detail should match the question.

The proximal-distal project uses a model ladder for the same reason. A double pendulum isolates coupling. A two-hand model exposes wrench redundancy. A flexible shaft adds storage. A moving base adds back-reaction. Spatial and full-body tiers add frames, contacts, and independent engine realizations. Each rung should answer a declared question and preserve common observables.

19.2 How the Mechanism Works

The bottom rung should be analytically transparent. Exact equations, limiting cases, and pointwise decompositions reveal why a mechanism appears. Higher rungs ask whether it survives added realism. If the sign reverses or effect vanishes, the model boundary becomes a scientific result rather than an embarrassment.

19.2.1 Common Observables Are the Handrail

Coordinates differ across tiers. A planar wrist angle has no one-to-one match in a spatial model with two wrists and a flexible shaft. Comparing raw coordinates would confuse model architecture with mechanism.

Instead, every tier reports a club wrench at a declared reference, interface power, club motion, relevant work integrals, and closure residuals. These are the handrail connecting rungs. Added internal detail can be reported without losing the shared comparison.

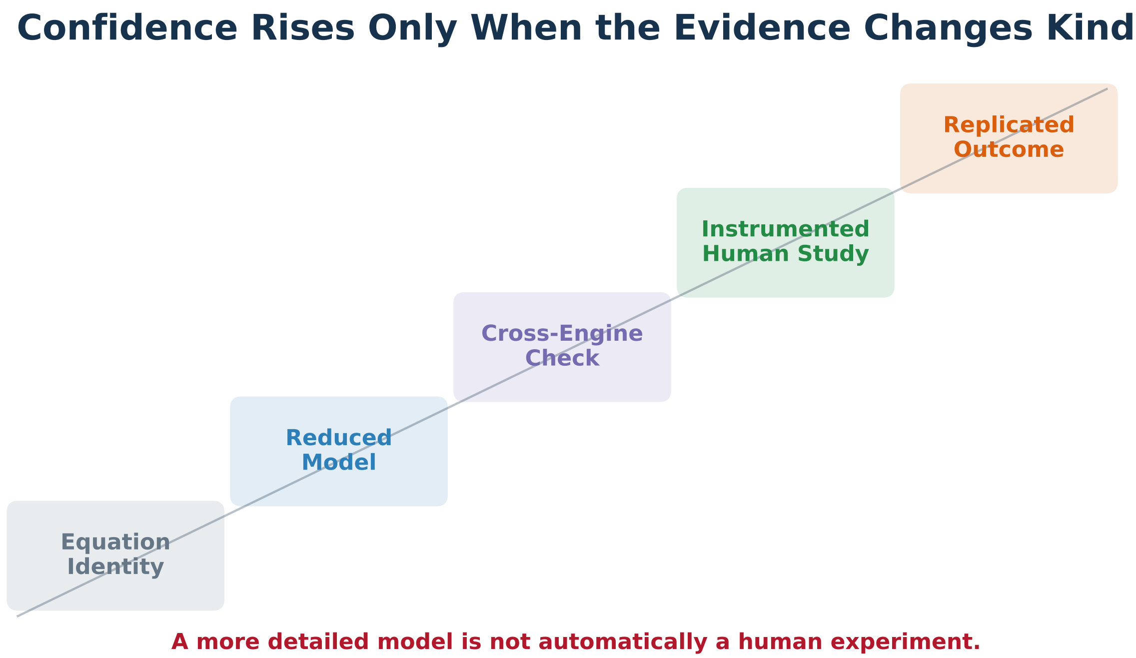

Figure 19.1: A Model Ladder Adds Two Hands, Flexibility, Moving Base, Spatial Motion, and Full-Body Structure While Keeping Common Wrench and Power Observables.

19.2.2 Complexity Must Pay Rent

Adding degrees of freedom increases parameter uncertainty and nonuniqueness. A full-body model can look lifelike while allowing many internal solutions. If it does not improve a relevant observable, falsifier, or uncertainty estimate, its complexity has not paid rent.

The distributed shaft is justified when higher modes change the impact-window wrench or energy beyond tolerance. The moving base is justified when back- reaction changes the mechanism. Spatial contact is justified when out-of-plane geometry or wrench components matter. The question comes first.

19.2.3 Surviving and Changing Claims

Some statements are structural. Action-reaction, wrench transformation, and work-energy closure should survive every tier. Some are model-specific: the exact duration of a negative couple, optimal torque onset, or numerical force magnitude can change with parameters and degrees of freedom.

The release matrix labels what each tier supports, omits, and must not claim. This prevents a result from climbing the ladder merely because a later model exists elsewhere in the repository.

19.2.4 Cross-Formulation Agreement

Inverse, forward, reduced, and spatial formulations can share a common state or observable. Agreement under independently implemented equations reduces the chance that one code path created the effect. Disagreement helps locate hidden assumptions.

The independently executed spatial contact test is especially useful because MuJoCo and Pinocchio do not share one dynamics implementation. Their agreement on bounded mechanism observables strengthens numerical credibility, though both still represent the same reduced physical idea.

19.2.5 The Ladder Ends at an Open Gate

No model rung becomes a human trial. The top computational tier still needs measured kinematics, bilateral hand wrenches, ground reactions, shaft behavior, and activation. The experimental protocol is not an optional future decoration; it is the next rung.

Model Result: The negative-couple and interaction-force mechanisms survive several rungs with changing quantitative values and explicit failures under geometry controls.

Human Evidence: Available golf measurements constrain motion and some net kinetics but do not yet supply the synchronized dataset needed to validate every common observable.

Optimization with higher-degree-of-freedom golf models illustrates how rapidly the design space grows as anatomical and club freedoms are added (McNally and McPhee 2018).

Practical Interpretation: Ask for the smallest model that can answer today's question and the next larger model capable of proving that the answer was not an artifact of the simplification.

19.2.6 Why No Single Model Is "the Model"

Every model answers a question by omitting something. A double pendulum isolates linked inertia. A two-hand constraint model exposes force allocation. A flexible shaft adds stored energy and phase. A whole-body spatial model adds moving base and ground interaction. A musculoskeletal model asks about candidate muscle allocations. None supersedes all earlier models; each changes the balance between interpretability, fidelity, and identifiability.

The ladder works when observables and interventions remain aligned across rungs. If every model uses a different definition of transfer, comparison is impossible. This project carries forward hand wrench, swing-normal couple, interface power, work, club state, energy closure, and declared counterfactuals.

19.2.7 Rung Zero: Algebra and Geometry

Before integrating motion, test wrench arithmetic, sign conventions, moment-arm dependencies, and coordinate transforms. Symbolic or static examples should recover known limits exactly. This rung catches errors that a complex animation can conceal.

It also states propositions clearly: collapsing hand separation collapses a pure differential couple; a consistent reference-point transform preserves wrench power; equal-and-opposite contacts can have zero net force and nonzero moment. These are mechanics identities, not human findings.

19.2.8 Rung One: The Planar Double Pendulum

The fixed-base two-link model tests interaction dynamics, carried motion, relative release, and torque timing. Its small state permits dense parameter sweeps, analytic checks, and transparent pointwise decompositions. It cannot represent bilateral hands, spatial axes, ground contact, or shaft deformation.

A result promoted from this rung should be qualitative and mechanism-focused. Exact speed gains or optimal milliseconds are provisional. Negative controls and sensitivity maps determine which signs or orderings are stable enough to carry upward.

19.2.9 Rung Two: Two Hands and a Moving Base

Add separated contact points and either prescribed or dynamic proximal motion. Now the model can reconstruct common and differential hand modes and test the geometric negative-couple hypothesis. A finite-mass base restores reaction from club to body. Contact remains idealized and anatomy generalized.

This rung asks whether the mechanism survives when the very feature it concerns —two hands—is represented explicitly. It also reveals nonuniqueness: the same club wrench can correspond to many internal contact allocations.

19.2.10 Rung Three: Flexible Club

Add one or more shaft modes, damping, and calibrated boundary coupling. Repeat the same counterfactuals while including modal state and energy. This rung asks whether rigid-club conclusions were artifacts of omitting deformation and whether negative coupling interacts with shaft return.

Modal convergence and independent beam checks matter. A single arbitrary spring can generate an attractive delay without representing a club. Parameters should come from measured or explicitly synthetic sources and remain qualified.

19.2.11 Rung Four: Spatial Whole-Body Dynamics

Add three-dimensional pelvis, trunk, shoulder/arm, hand, and club motion with ground contact. Spatial frames permit out-of-plane moments and full wrenches. The number of parameters and controls grows sharply, so identifiability becomes a central result rather than an inconvenience.

Independent engine agreement is valuable here. Matching trajectories is not enough; constraint reactions, energy, and wrench projections should also agree. A mechanism surviving only one software stack remains suspect.

19.2.12 Rung Five: Biological Allocation

A musculoskeletal or activation-aware model proposes how generalized loads might be shared among muscles and passive tissues. It can study co-contraction, stiffness, and internal load, but it needs EMG, strength, anatomy, and optimization assumptions. Multiple allocations usually fit the same motion.

This rung should not retroactively turn earlier generalized arm and wrist drivers into named muscles. Instead it tests candidate mappings and reports their nonuniqueness. Scapular action, in particular, requires adequate kinematics and measurement-aware shoulder-girdle representation.

19.2.13 Promotion and Demotion Rules

A claim is promoted when it passes implementation verification, negative controls, uncertainty tests, and the next rung's relevant physics while retaining a coherent observable prediction. It is demoted when sign or mechanism depends on a removed assumption, fails closure, or cannot be distinguished with available measurements.

Demotion is not failure of the project. It locates a boundary: "this effect is specific to a fixed-base rigid planar model" is scientifically useful. The reader can then see what remains established, what is model-conditional, and what human evidence still owes.

19.2.14 Cross-Rung Tables and Figures

For each mechanism, a comparison table should list degrees of freedom, contact model, shaft model, controls, identified parameters, pass/fail gates, and common outcomes. Figures should overlay normalized time histories with consistent signs, not juxtapose unrelated axes. Differences in magnitude should remain visible rather than normalized away when magnitude is the question.

Machine-readable manifests make this comparison reproducible. Each row links to input, output, code commit, and generated figure. Reviewers can rerun one rung or replace it with an independent implementation without adopting the whole software ecosystem.

19.2.15 Deciding When Complexity Is Worth It

Add a degree of freedom or state when it changes a target observable, explains a structured residual, enables a discriminating experiment, or removes an unacceptable assumption. Do not add complexity merely for visual realism. Unidentifiable complexity can make a model less falsifiable by allowing it to fit anything.

The final modeling product should therefore remain modular: a transparent core, optional higher-fidelity rungs, shared contracts, and explicit qualification. That architecture invites criticism and extension as an open resource rather than presenting one branded engine as final authority.

19.2.16 Reader Lab: Write a Promotion Matrix

Choose one claim, such as a late negative swing-normal contact couple. Across rows list algebraic geometry, planar double pendulum, two-hand moving base, flexible shaft, spatial whole-body, and biological allocation. Across columns list sign, onset, duration, magnitude, work, outcome effect, negative controls, uncertainty, and human evidence. Populate every cell with result, not applicable, or unresolved.

The matrix makes a common error obvious: a claim may be well verified in early rungs but unresolved in humans. It also reveals where complexity changes the result. Link each cell to machine-readable evidence and tests.

Now define promotion criteria before looking at the next rung. Perhaps sign must hold in 95 percent of a declared ensemble and geometry controls must pass, while magnitude can vary. If criteria fail, narrow the claim. This avoids silently moving the goalposts as realism increases.

19.2.17 Independent Extension

Invite a second implementation to reproduce one rung using the published contracts. If it disagrees, preserve both outputs and reconcile specification, frames, and numerics. The goal is not to defend an engine. It is to discover which physical conclusion survives independent expression.

A modular open resource makes such extension inexpensive. Shared schemas and benchmarks let a contributor add a contact or anatomical rung without rewriting the manuscript's conceptual core.

19.2.18 A Claim Can Live at More Than One Rung

The identity that wrench power is reference-point invariant belongs to the mechanics rung and should hold everywhere. The finding that a particular negative-couple interval appears belongs to specific model rungs. The hypothesis that golfers use it belongs to the human-evidence rung. Separating them prevents a human null result from invalidating an identity or a verified identity from being offered as human validation.

Each public statement should therefore carry rung and status. Readers can see which parts are durable foundations and which remain contingent. New evidence updates the appropriate layer without rewriting the entire framework.

19.2.19 Completion Is a Maintained Ladder

The ladder is complete enough for release when every rung has a specification, verification, shared observables, limitations, and a documented route to the next test. It does not require maximal anatomical detail. Future rungs can be added when evidence and questions justify them.

Maintaining simple benchmarks alongside rich models prevents regression and keeps the project teachable. The lay book, technical monograph, simulator, and evidence register should all reference the same ladder identifiers.

19.3 Where the Picture Breaks

The map analogy suggests clean nested scales. Model families are not always nested; a spatial engine may use different contact or damping laws rather than simply adding detail. Common observables and negative controls are needed to compare them fairly.

More agreement also does not guarantee truth if all tiers share a wrong premise. Independent measurement remains decisive.

Go Deeper: Explore the monograph's common-observable ladder, model completion matrix, and reviewer workbench.

Chapter 20

From a Plane to Three Dimensions and Independent Engines

20.1 A Concrete Picture

A shadow puppet can reveal a hinge mechanism while hiding motion toward the wall. A planar golf model is a carefully chosen shadow. It can expose swing- normal force and torque mechanisms, but it cannot see out-of-plane clubface moments, changing swing plane, or spatial joint axes.

Moving to three dimensions is not merely drawing the same links with perspective. Rotations no longer commute, moments require declared frames, contacts have six- axis wrenches, and constraints can contain new internal modes.

20.2 How the Mechanism Works

In a plane, one angle and one signed moment describe each hinge. In space, body orientation belongs to the rotation group $SO(3)$. Angular velocity is a vector, and a moment's components depend on frame. Applying rotations in different orders can yield different orientations.

20.2.1 Spatial Wrenches Preserve the Physical Contract

A three-dimensional wrench combines three force and three moment components. When moved to another reference point, its moment changes by the cross product of offset and force. Power combines force with linear velocity and moment with angular velocity. Consistent spatial algebra preserves power under frame change.

This frame-explicit contract is the bridge from the planar negative couple to a spatial test. The swing-normal component can be compared while additional components reveal whether the mechanism creates undesirable twist or bending.

20.2.2 Full Body Adds Redundancy

A spatial full-body model has many coordinates capable of producing similar club motion. Inverse dynamics may estimate a net generalized-force vector for a prescribed trajectory, but two-hand contact

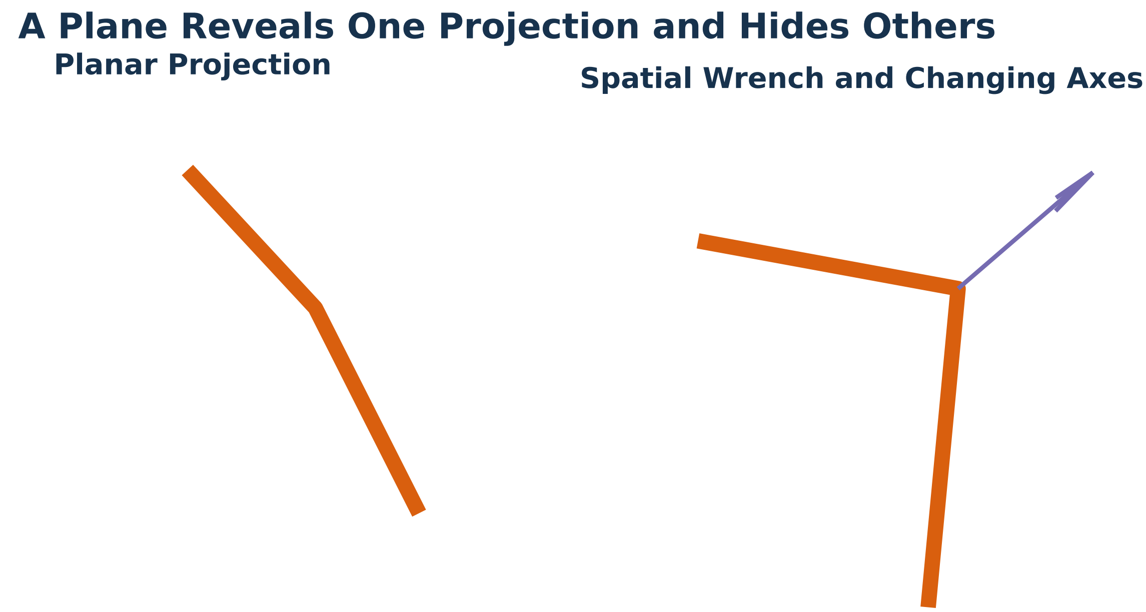

Figure 20.1: A Planar Shadow Captures Swing-Normal Mechanics While a Spatial View Adds Out-of-Plane Force, Twist, and Changing Axes.

and muscle redundancy remain. More coordinates do not automatically identify more biology.

The common-state spatial model therefore emphasizes action-reaction, wrench closure, and matched interventions rather than claiming a unique human strategy. It reports what survives and what remains open.

20.2.3 Contact Closure Comes Before Contact Dynamics

A constraint Jacobian can have full row rank even when the points it would constrain are far apart. The project tested that distinction with six deterministic subject-scaled engineering profiles, three grip spans, and 61 prescribed states per case. The anatomical hand points miss the declared grip points by 0.171–0.616 m, and none meets the registered 5 mm tolerance.

The same open configurations still produce a rank-six bilateral contact Jacobian. That is not a contradiction. Rank asks whether six local correction directions are available; closure asks whether a correction is still required. Calling a rank test a contact-feasibility result would therefore hide the most important geometric failure.

This adverse result narrowed the next model rather than weakening the scientific program. A bounded subject-scaled solver then closed both hands on the grip while respecting joint limits and collisions. It held the club at 13 prescribed phase samples and closed **234 of 234** profile–span–phase cases, with no contact error above 1.16×10^{-10} m. A deliberately unreachable 2.0 m grip span failed, preserving a negative control.

That success is a reduced-tree necessary condition, not proof that a person can adopt the poses. The limits are broad engineering guards, collision clearance uses coarse spheres, and the model omits scapular motion, forearm rotation, multi-axis wrists, fingers, and distributed grip contact. The spatial tier still cannot answer whether a passive mechanism reduces timing demand, creates useful slack, improves delivery, or specifies a human technique. Subject-specific anatomy and calibrated compliant forward contact are the next gates.

20.2.4 Scapular Mobility Changes Reachability Without Identifying Strategy

The next paired screen changes only one structural assumption. It holds the trunk and club pose fixed, then compares the original fixed shoulder centers with shoulder centers that can move on a declared thoracic ellipsoid. Four screening coordinates per side represent protraction, elevation, upward rotation, and winging. This is a reduced geometry experiment, not a digital human shoulder.

The fixed branch closes **0 of 54** registered states. The mobile branch reaches the residual tolerance in **31 of 54**, but only **16 of 54** also receive a successful optimizer termination. Twenty-eight states touch a declared bound, and an intentionally impossible 2.0 m grip span remains open. Mobility clearly changes modeled reach, but unresolved cases may reflect range choice, omitted coordinates, or numerical termination rather than human infeasibility.

There is a second warning. Both contact Jacobians retain rank six, while local coordinate nullity rises from two to ten. Many scapular and glenohumeral coordinate changes can therefore satisfy the same bilateral contact task. Contact geometry alone cannot tell a reviewer which internal allocation a person used or should use.

The scientific next step is not to rename these coordinates as muscles or coaching actions. It is to repeat the comparison with a validated articulated shoulder, subject-specific bone geometry where governed, calibrated distributed grip contact, and paired forward dynamics. Only then can force, work, passivity, delivery, or human-strategy hypotheses be tested.

20.2.5 Independent Engines Test Implementation Dependence

Running the same conceptual experiment in two engines is useful only if the implementations are genuinely independent and observables are transformed to a common convention. Matching screenshots prove little. Matching trajectories, wrenches, powers, constraint residuals, and negative controls is stronger.

MuJoCo and Pinocchio differ in their numerical and contact machinery. The project's finite-mass hand-carriage experiment uses paired compliant interfaces and no direct club actuation in both. Its same-state driver killswitch and moment- arm controls make the comparison mechanistic.

20.2.6 Agreement Has a Boundary

If two engines agree on a reduced spatial mechanism, code-specific error becomes less likely. Shared parameter errors, oversimplified contact, and missing anatomy remain possible. Cross-engine agreement is verification evidence, not human validation.

20.2.7 Three-Dimensional Measurement Is Hard

Marker-based motion capture suffers soft-tissue artifact, especially around the scapula and forearm. Joint centers and axes are estimated. Clubface orientation requires careful rigid-body tracking. Bilateral grip sensors must be registered to the club frame. These uncertainties should enter the model-data comparison.

Model Result: A reduced spatial mechanism retains the late negative contact couple across two independent engine realizations for the declared interval and passes sign-changing controls.

Human Evidence: Three-dimensional golf motion is clearly not planar, but existing heterogeneous datasets do not validate the specific spatial contact mechanism (Meister et al. 2011; Joyce et al. 2013; McPhee 2022).

Hypothesis: The planar swing-normal couple is one projection of a spatial wrench pattern that should be measurable without requiring a unique muscle allocation.

Practical Interpretation: A planar model can explain a shadow mechanism; a spatial model must show which parts remain when the lights move.

20.2.8 What the Plane Captures

A planar model captures rotation in one chosen swing plane and forces projected into it. It can represent link inertia, relative release, radial/tangential reaction, and an out-of-plane moment. Because signs and geometry are easy to see, it is an excellent place to derive and falsify the basic negative-couple mechanism.

The plane is not a claim that a golf swing is flat. It is a controlled slice. The key question is whether a spatial extension retains the corresponding physical projection when out-of-plane motion and extra axes are admitted.

20.2.9 What Three Dimensions Add

Each rigid body now has spatial orientation and angular velocity. The club's moment is a vector. Hand forces and moments have six components per contact. Swing-plane normal can vary with time and must be defined operationally. Forearm rotation, wrist deviation, shaft droop, torso side bend, and shoulder-girdle motion introduce pathways absent from the plane.

These freedoms can redistribute a planar effect rather than merely add to it. A moment that looked entirely negative in one axis may split across changing axes. The invariant quantity is the complete wrench and its power; the chosen swing-normal projection remains useful if its construction is reported and tested for sensitivity.

20.2.10 Frames Must Be Registered

Laboratory, pelvis, trunk, arm, hand, club, sensor, and swing frames all move differently. Every spatial wrench transform includes rotation and moment shift. Angular velocities require consistent representation; Euler-angle derivatives are not the angular-velocity vector. These are common sources of plausible but wrong signs.

Frame tests use known loads and orientations, action–reaction at contacts, reference-point invariance of power, and round-trip transforms. Coordinate conventions should be machine-readable. A reviewer should be able to reproduce the swing-normal axis from released kinematics.

20.2.11 Defining the Late Impact Zone

"Near impact" can be a clock window, a range of club position, a state-space region, or a phase normalized between events. A spatial study should declare the primary definition and test alternatives. If the negative-couple interval appears only under one post hoc window, confidence is low.

State-based regions may include clubhead height, hand path, shaft orientation, and arm–club geometry. They allow comparisons across tempo but can introduce selection bias if tuned on outcomes. Freeze them before participant holdout.

20.2.12 Independent Engines

Implement the same spatial mechanism in two dynamics engines or formulations. Share model parameters, initial state, control histories, and output schema. Compare trajectories, hand wrenches, constraints, energy, and the projected couple within tolerances. Differences should be investigated, not averaged.

Independence is strongest when implementations do not share derivative code or hidden convention helpers. Agreement then reduces the chance that one algebraic or frame error created the result. The released comparison should include both successful agreement and known deviations.

20.2.13 A Spatial Geometry-Control Suite

Reverse the defined swing-normal axis and verify only the reported projection changes sign, not physical power. Swap hand positions in the model and verify the differential couple responds. Collapse separation, align forces with the separation vector, and rotate the swing plane. Perturb sensor-frame orientation to quantify sign confidence.

These tests distinguish coordinate behavior from physical mechanism. If a purported negative couple disappears whenever the reporting plane shifts a few degrees, its interpretation should include that uncertainty rather than a binary label.

20.2.14 Moving Anatomy Without Overclaiming It

Spatial arm segments permit richer kinematics, but a ball joint at the shoulder does not represent scapulothoracic motion or muscle recruitment automatically. Marker-based shoulder centers and scapular orientation have substantial measurement uncertainty. A generalized shoulder driver remains an abstraction.

The model can predict a required hand wrench or joint-level generalized load. Mapping that to scapular retraction, protraction, or named muscle action requires additional kinematics, EMG, and a biological model. Keeping that boundary explicit protects the very hypothesis the work aims to test.

20.2.15 Quantities That Should Survive the Extension

Whole-system energy balance, contact action–reaction, wrench power invariance, and geometric control responses should survive exactly or within numerical tolerance. Qualitative timing and sign of the proposed contact contribution may survive over a declared region. Exact optimal controls and magnitudes are expected to change.

The spatial model earns its complexity if it makes new falsifiable predictions: out-of-plane force components, axis-dependent couple timing, or conditions under which the planar mechanism fails. If it merely draws a more realistic golfer while hiding the causal tests, it has moved away from science.

20.2.16 Human Validation Path

Use synchronized full-body and club motion, bilateral grip wrenches, separate force plates, and shaft deformation. Calibrate every frame and report soft- tissue and sensor uncertainty. Evaluate participants held out from parameter tuning. Compare predicted three-dimensional wrench trajectories, not only clubhead speed.

Residuals should be inspected by phase and direction. A systematic out-of-plane residual may identify missing wrist, shaft, or shoulder-girdle physics. Updating the model in response should be declared as a new rung and retested on untouched participants.

20.2.17 Reader Lab: Rotate the World

Take one spatial state and rigidly rotate every position, orientation, force, moment, and gravity vector into a new laboratory frame. Recompute dynamics. The rotated physical trajectory, energy, and wrench power should match. A quantity tied to the declared swing frame should transform predictably. Failure reveals a frame-dependent implementation error.

Next project the same contact moment onto several plausible swing-plane normals. Report sign confidence and angular sensitivity. If all definitions agree over the late interval, the negative-couple claim is robust to plane construction. If they disagree, publish the dependence and prefer the complete vector result.

Finally compare two independent spatial engines on geometry-reversal and hand- separation controls. Match initial state and input at high precision. Differences in constraint stabilization or quaternion handling should remain below declared observable tolerances.

20.2.18 Spatial Visualization Without Illusion

A three-dimensional rendering can make a model look biologically authoritative. Include visible joint definitions, hand contact frames, force-vector scale, uncertainty, and model-rung label. Let readers rotate the scene and inspect the numeric state. Do not use a detailed human mesh to imply anatomical fidelity not present in the equations.

The best spatial figure pairs an intuitive view with invariant plots: wrench power, projected couple with axis sensitivity, and closure residual. Beauty should make the mechanism easier to audit, not harder to question.

20.2.19 A Minimal Spatial Report

Publish body and joint frames, orientation representation, state definition, inertias, constraints, controls, solver, and swing-plane construction. Report complete three-dimensional hand wrenches before selected projections. Include energy, constraint, and engine-agreement residuals. Provide geometry-control animations with fixed vector scales and downloadable data.

For the negative couple, show the moment vector in club, laboratory, and swing frames, the angle between vector and plane normal, and sign probability under frame uncertainty. Report the interval by both time and state. This makes a spatial claim reviewable rather than visually persuasive only.

20.2.20 When the Planar Result Fails in Space

Failure can occur because out-of-plane motion redirects the contact wrench, because a different axis dominates, because hand geometry changes, or because the planar constraint created an artificial reaction. Diagnose which term changes. A spatial failure is not proof that coupling is irrelevant; it narrows the specific projection or validity region.

The revised explanation might say that a negative total swing-normal couple is not robust, while one component or state-dependent regulation effect remains. Such refinement is exactly why the model ladder exists.

20.2.21 Accessibility of a Three-Dimensional Result

Pair an interactive scene with two-dimensional explanatory slices and textual alt descriptions. A phone reader should understand hand positions, arrows, and sign without rotating a model. A specialist can open the interactive view and data. Providing both prevents visual sophistication from becoming an access barrier.

20.2.22 Chapter Checkpoint

The spatial extension succeeds when it preserves invariant balances, survives frame and engine tests, and either confirms or precisely narrows the planar mechanism. More degrees of freedom alone are not progress. New, measurable predictions and exposed failure regions are.

For a reviewer, the full wrench vector remains primary. A convenient negative projection should never conceal uncertainty in the axis used to define it.

20.3 Where the Picture Breaks

The shadow analogy makes spatial extension look visually obvious. In fact, coordinate conventions, rotation representations, and contact frames can create subtle sign errors. Every comparison needs automated transformation and power checks.

Engine agreement can also become false reassurance if one result is tuned to the other. Parameters and tolerances should be registered before comparison, with held-out cases and negative controls.

Go Deeper: Read the spatial common-state formulation, the independent forward-contact chapter, and the project's reproduction boundary.

Chapter 21

Sensitivity and Identifiability

21.1 A Concrete Picture

Taste a soup and notice it is salty. That observation may be sensitive to the amount of salt, but it cannot tell whether salt was added directly, broth was reduced, or a salty ingredient was included. Several recipes can produce the same taste. Sensitivity asks how the taste changes when an ingredient changes. Identifiability asks whether tasting the soup lets us recover the recipe.

Golf models face the same distinction. Club speed may be sensitive to shoulder torque and wrist timing. Yet one speed trace may not uniquely identify those inputs because shaft stiffness, hand path, damping, contact allocation, and inertial parameters can compensate.

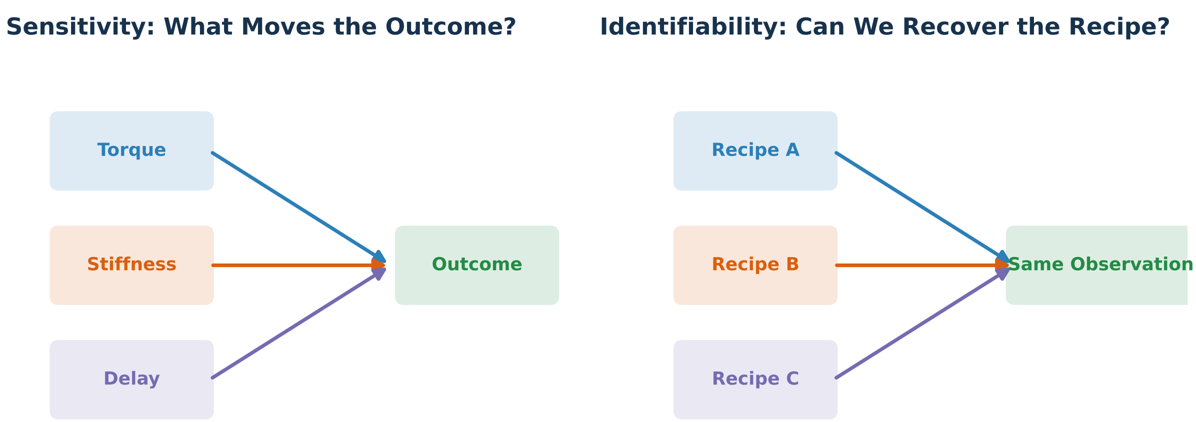

Figure 21.1: Sensitivity Shows Which Inputs Move Each Outcome, While Identifiability Shows Which Inputs Can Be Recovered From the Available Observations.

21.2 How the Mechanism Works

Local sensitivity changes one parameter slightly near a reference and estimates an outcome derivative. Global sensitivity varies many parameters across declared ranges and apportions outcome variance. A parameter can be locally quiet yet important through interactions elsewhere in the range.

21.2.1 Structural and Practical Identifiability

Structural identifiability asks whether perfect, noise-free observations under the model could uniquely recover parameters. Practical identifiability includes finite data, noise, limited excitation, and correlated effects. A parameter may be structurally identifiable and practically hopeless.

The coupled study varies 12 inputs simultaneously but observes six summary quantities. The full parameter vector cannot be identified from that information. This is not a solver failure. It is an information limit.

21.2.2 Hidden Hand-Force Modes

Even perfect net club wrench leaves an individual-hand-force mode unresolved. No amount of optimizing club trajectory can recover a quantity absent from the observations. Bilateral sensors add the missing information; a preferred regularizer merely selects one possible allocation.

21.2.3 Sensitivity Is Outcome-Specific

An input can strongly affect speed and weakly affect face/path, or vice versa. Ranks should therefore be reported by outcome. Combining everything into one score hides tradeoffs and lets arbitrary weights select the answer.

21.2.4 Uncertainty Should Travel to Conclusions

Parameter ranges come from measurement, literature, or explicit synthetic design. Samples propagate those ranges through the model. Reporting medians, tails, and failure rates is more honest than one nominal trajectory. Held-out samples test whether a chosen program generalizes beyond the cases used to select it.

21.2.5 Identifiability Guides Measurement

The value of analysis is not only declaring uncertainty. It identifies which new observations would separate competing explanations: bilateral hand wrenches, shaft strain, activation timing, or a richer time series instead of six summaries. Experiment design can target sensitive and discriminating states.

Model Result: The 12-input coupled design shows practical nonidentifiability, one unresolved two-hand force mode, and outcome-dependent sensitivity.

Human Evidence: Methodological heterogeneity and participant variability in golf research make parameter recovery more difficult, not less (McPhee 2022).

Hypothesis: Bilateral wrench and shaft measurements will distinguish arm- wrist allocation mechanisms that club motion alone cannot identify.

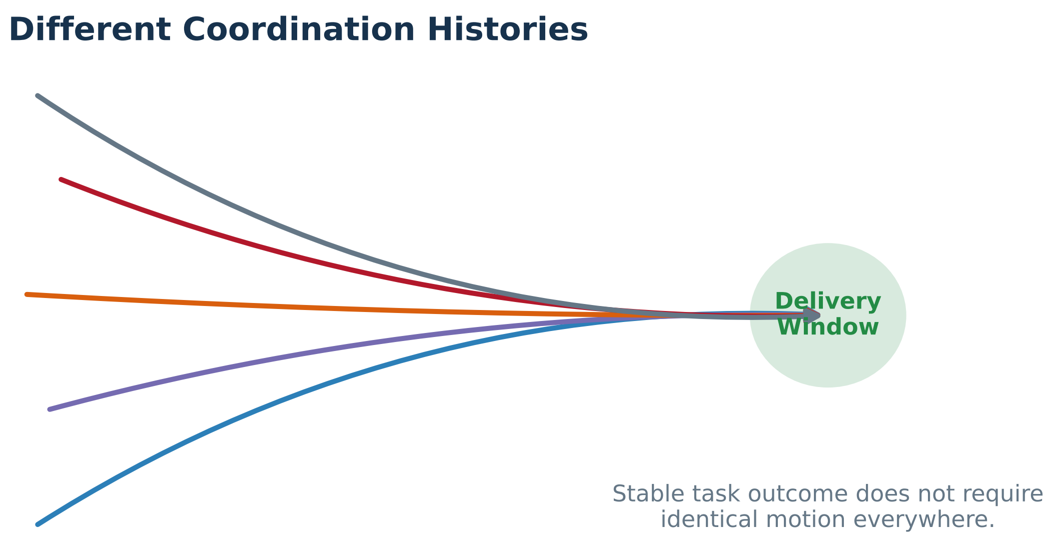

Figure 21.2: Many Internal Paths Can Reach One Club-Level Outcome, Which Is Why Motion Alone Cannot Recover the Hidden Allocation.

Practical Interpretation: A result can change strongly when a parameter changes while the observed result still cannot tell us which parameter changed.

21.2.6 Start With the Question, Not the Method

Sensitivity asks what changes when an input changes. Identifiability asks what inputs can be learned from observations. Optimization asks which inputs best meet an objective. Calibration asks which inputs make a model resemble data. These questions use related mathematics but produce different claims. A parameter can be influential yet unidentifiable, or identifiable yet irrelevant to the output of interest.

For the proximal-distal product, write each question explicitly: Which inputs alter late hand couple? Can bilateral wrench data distinguish arm and wrist allocation? Which timing policy is robust across uncertain inertia? Can shaft stiffness be estimated from strain and motion? This prevents one global ranking from being treated as an answer to all four.

21.2.7 Local Sensitivity

Near a nominal parameter vector, perturb one input by a small amount and estimate the derivative of an outcome. Automatic differentiation, analytic sensitivities, or carefully checked finite differences can compute it. Scaling matters: a derivative per kilogram cannot be compared directly with one per millisecond. Use nondimensional or uncertainty-scaled effects.

Local derivatives reveal sign and steepness around one case. They can miss thresholds, sign reversals, and interactions elsewhere. A dead-zone transmission model is a clear example: sensitivity may be nearly zero while disengaged and large near engagement. Multiple nominal states and directional perturbations are needed.

21.2.8 Global Sensitivity

Define justified ranges and joint distributions, sample them, and propagate each case through the model. Variance-based indices, elementary effects, or surrogate models can identify main and interaction effects. The result belongs to the chosen range and distribution. A broad arbitrary range can exaggerate a parameter; a narrow convenient range can hide it.

Outcome-specific reporting is essential. Arm inertia might dominate terminal speed variance while hand separation dominates couple magnitude and command rise time dominates face/path sensitivity. A single importance score would erase the design tradeoff.

21.2.9 Structural Nonidentifiability of Two Hands

Suppose only the net club wrench is observed. Adding one internal hand-force mode and subtracting it from the other can preserve the same resultant. No noise-free club trajectory can recover that hidden mode without additional assumptions. This is structural nonidentifiability, not a shortage of optimizer effort.

Regularization can choose a minimum-force or minimum-effort allocation, but the choice is a prior. Report it as one candidate. Bilateral sensors add observables that collapse much of the null space and transform the scientific question from speculation to estimation.

21.2.10 Practical Identifiability in a Rich Model

Even theoretically identifiable parameters can be strongly correlated in finite noisy data. A more flexible shaft can mimic timing delay; a different hand path can mimic torque onset; inertial error can be absorbed by estimated control. Posterior distributions, profile likelihoods, or bootstrap intervals reveal these ridges better than one best fit.

Excitation matters. If all recorded swings occupy nearly the same state region, the data cannot distinguish mechanisms that diverge elsewhere. Safe equipment or task manipulations can increase information by changing hand separation, shaft response, tempo, or load geometry.

21.2.11 A Worked Ambiguity

Imagine terminal speed rises one metre per second when either wrist-torque onset moves five milliseconds or shaft stiffness increases ten percent. One speed number cannot tell which changed. Add a shaft-strain waveform: stiffness affects modal phase differently from command timing. Add bilateral wrench: onset changes contact history. The extra observations make candidate explanations separable.

Now suppose both candidates still fit because motion filtering uncertainty shifts phase. The correct result is a joint confidence region, not an arbitrary winner. Experiment design should target a phase or manipulation where their predictions separate beyond measurement uncertainty.

21.2.12 Sensitivity of a Sign Claim

For the late negative couple, calculate not only mean magnitude but probability of negative sign across parameter and measurement uncertainty. Report onset and duration distributions. Identify the parameters most associated with sign failure. Geometry reversal and zero-separation controls should remain decisive across the ensemble.

If sign confidence is only slightly above chance, use qualified language. If sign is robust but magnitude uncertain, the mechanism claim can be stronger than the performance claim. Distinguishing those levels is more informative than one binary validation stamp.

21.2.13 Participant-Held-Out Tests

Fit or select policies on one set of participants and freeze them before evaluation on another. Split by person, not swing, because repeated trials from one golfer share anatomy and strategy. Report individual outcomes and population distribution. A model that predicts the average but fails most individuals is not an individualized product.

When data are limited, leave-one-participant-out evaluation and hierarchical models can help, but uncertainty will remain wide. Honest width is preferable to leakage that creates impressive but non-reproducible accuracy.

21.2.14 Measurement Value as a Design Output

Use the sensitivity and identifiability analysis to rank candidate sensors by which hypotheses they discriminate. Bilateral wrench may be essential for hand allocation; shaft strain for elastic state; high-quality shoulder-girdle motion for scapular geometry; EMG for activation timing; perturbation response for effective stiffness.

The best next measurement is not always the most precise. It is the one that separates live hypotheses at acceptable cost and burden. This turns uncertainty analysis into an experimental roadmap rather than a disclaimer.

21.2.15 What to Publish

Release parameter definitions, units, ranges, distributions, correlation assumptions, sample design, convergence diagnostics, sensitivity estimator, outcome definitions, and raw indices. Include failure cases and plots showing nonlinear response surfaces. Link every summary back to a machine-readable table. Reviewers can then test alternate priors without rerunning the entire argument from prose.

21.2.16 Reader Lab: Design One Discriminating Measurement

Select two parameter or mechanism combinations that fit current club motion. Simulate candidate observations: bilateral wrench, shaft strain, ground reaction, or EMG timing. Plot where predictions differ relative to expected sensor noise. Choose the observation and phase with the largest reliable separation and design a bounded trial around it.

After collecting data, evaluate the predeclared discriminator on held-out trials. Do not refit both models until they agree and then claim neither was rejected. If data fall between predictions, quantify whether both are wrong or uncertainty was understated.

Repeat the exercise for the arm/wrist allocation extremes. Club speed may be nearly identical while differential hand force and transition stiffness differ strongly. This demonstrates how identifiability analysis directly determines instrumentation priorities.

21.2.17 Communicating an Unidentifiable Result

Do not say "the parameter could not be estimated" and stop. Show the family of values or combinations consistent with data, the observables they predict, and the measurement that would shrink the family. If a practical conclusion remains constant across it, that conclusion may still be robust. If the conclusion changes, the product should withhold recommendation.

This is one of the clearest ways to expose falsifiability: uncertainty is represented as competing predictions a future experiment can separate.

21.2.18 A Minimum Identifiability Report

List every estimated parameter and observed channel. State structural nonidentifiabilities, priors, bounds, correlations, noise model, calibration data, and holdout design. Plot posterior or profile relationships, not only standard errors around one optimum. Show predictive coverage on untouched data.

For hand allocation, display the null-space dimension with and without bilateral sensing. For preload, show correlation among transmission gap, activation delay, shaft dynamics, and onset. For robustness, show which parameter uncertainty changes policy ranking. These plots reveal what the current product can and cannot personalize.

21.2.19 When More Data Do Not Help

Collecting more repetitions of the same uninformative motion reduces random noise but may not separate structurally equivalent mechanisms. Richer excitation or a different sensor is needed. Conversely, adding many noisy sensors can increase processing assumptions without adding discriminating information.

Experiment design should therefore maximize expected distinction between live hypotheses under realistic noise and burden. The result is a ranked measurement plan tied directly to completion of model claims.

21.2.20 Conservative Product Behavior

When inputs are unidentifiable and conclusions vary across compatible values, the interface should say so and avoid a unique strategy result. It can show the family, sensitivity, and next useful measurement. If all compatible values lead to the same qualitative conclusion, it can report that robustness while retaining magnitude uncertainty.

21.2.21 Chapter Checkpoint

An influential parameter is not necessarily recoverable, and a fitted parameter is not necessarily causally important. Show both maps. When ambiguity affects a decision, withhold the decision and name the measurement that would resolve it.

21.3 Where the Picture Breaks

Soup ingredients are independent items; mechanical parameters interact through equations and may change meaning across models. Sensitivity indices depend on chosen ranges and distributions. Narrow ranges can make an important mechanism look unimportant.

Identifiability within a wrong model is also not truth. Data may precisely select a parameter that compensates for omitted physics. Residual structure and cross- model validation remain essential.

Go Deeper: Read the monograph's uncertainty and identifiability chapter and the machine-readable study outputs.

Chapter 22

Timing Is a State Question, Not Just a Clock Question

22.1 A Concrete Picture

A cook does not remove bread from an oven simply because twelve minutes passed. The timer is useful, but color, temperature, and structure tell whether the loaf reached the intended state. Two ovens or loaf sizes reach that state at different times.

Swing instructions often use clock language: start the release later, apply a torque earlier, wait until a position. A model can define these precisely, but the useful trigger may be a club angle, relative velocity, hand-speed event, shaft state, or predicted time to impact rather than elapsed milliseconds.

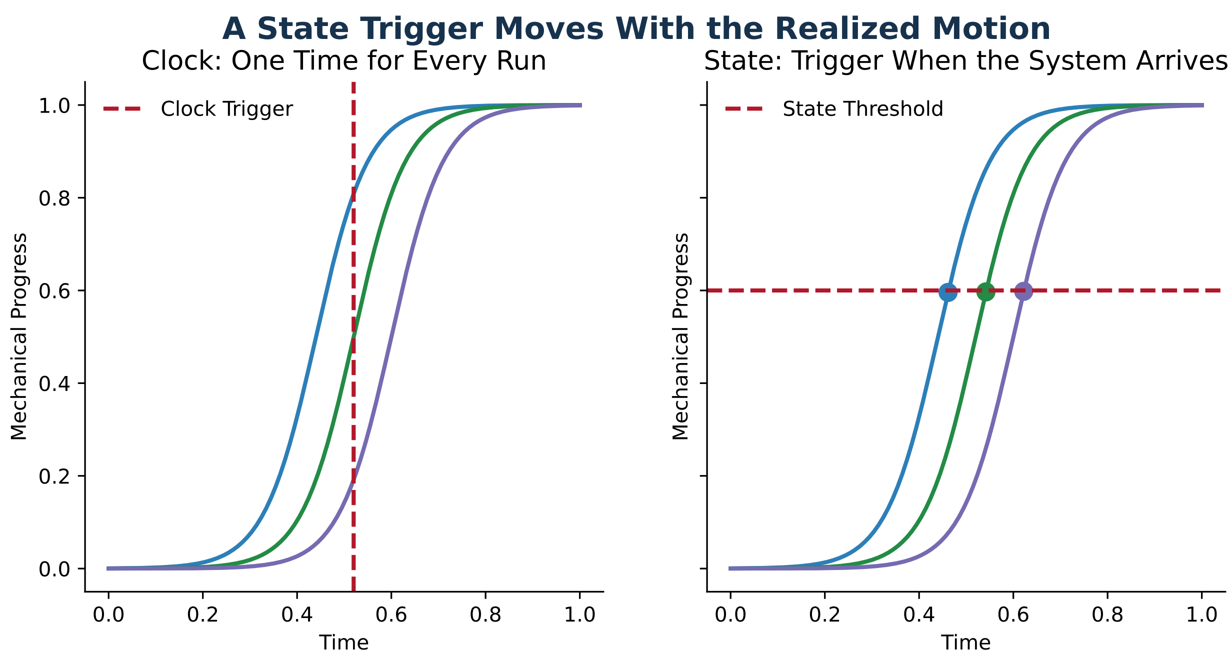

Figure 22.1: Clock-Time Triggers and State-Based Triggers Select Different Command Onsets When Two Swings Progress at Different Rates.

22.2 How the Mechanism Works

A torque program needs amplitude, sign, onset, rise time, duration, and shutdown. An onset can be fixed in time or defined by an event function $g(\mathbf{x}) = 0$. Event-based timing can align mechanisms across parameter variations, but the event must be observable and chosen before outcomes.

22.2.1 Early Drive and Retain-Then-Drive

The simple comparison applies positive wrist torque immediately after transition or uses zero/negative early torque followed by positive late torque. Optimization over onset shows late drive can outperform early drive under declared actuator bounds. This supports a timing mechanism, not a universal delay number.

The early opposing program tests a different idea: retaining state and preload may improve robust speed under delay. In the held-out ensemble it improves one lower-tail speed comparison while worsening face/path proxy. Timing has multiple objectives.

22.2.2 Command Timing Is Not Force Timing

Activation and transmission dynamics delay force after command. A 35 ms rise time means the actuator does not jump to its requested level. A preloaded channel may transmit immediately even when desired magnitude changes. Complete sign reversal may pass through low force. Distinguishing command, internal state, transmitted torque, and resulting motion prevents false timing claims.

22.2.3 Impact Definition Changes Optimization

If impact is defined by a club-angle crossing, a strategy altering trajectory also alters impact time. A fixed terminal clock time can compare different club positions. Robust studies repeat plausible impact criteria and reject schedules that never reach a valid event.

22.2.4 State Feedback Versus Open-Loop Schedules

An open-loop program plays the same command versus time regardless of deviation. A feedback strategy responds to state. Humans use sensory and predictive control, though delays limit rapid correction near impact. The model workbench lets readers compare clock and state triggers without calling either a complete neural controller.

Model Result: Preferred onsets move with parameters, rise time, and objective; the qualitative late-drive ordering survives declared command filters.

Human Evidence: Observed timing varies across golfers and measurement definitions. Kinematic events do not directly reveal command onset.

Computer simulations of delayed release show why timing claims must be stated against a declared model, objective, and state (Sprigings and MacKenzie 2002).

Hypothesis: Preparation and early downswing arrange a state in which late coupling and distal power become favorable; event-based triggers may generalize better than fixed clock delays.

Practical Interpretation: “Late” should mean late relative to a mechanical state or event, not a mystical timestamp copied across swings.

22.2.5 Clock Time Is Easy to Measure and Easy to Misread

Researchers commonly express events as milliseconds before impact or percentage of downswing. This supports averaging, but impact alignment stretches or compresses mechanically different trajectories onto one axis. Two golfers at the same normalized time may have different arm–club angles, velocities, shaft states, and force directions.

Clock timing remains useful when fully defined. Report event detection, normalization, uncertainty, and absolute duration alongside normalized phase. Do not treat a population mean as the command time used by any participant.

22.2.6 State Timing

A state trigger activates when a measurable condition is met: an arm–club-angle range, sign change in relative angular velocity, handle radial velocity, shaft mode phase, or interaction-term threshold. Such triggers can adapt to tempo and parameter differences. They also risk being difficult to measure or overly tuned.

Candidate triggers should come from mechanism, then be frozen before held-out testing. A trigger based on a hidden model variable may be useful for simulation but cannot guide an experiment until an observable proxy is validated.

22.2.7 Command, Force, Motion, and Outcome Events

Neural command onset, EMG onset, muscle force rise, joint torque, hand wrench, shaft response, angular acceleration, velocity peak, and impact occur at different times. Delay and filtering separate them. Seeing a club event does not reveal when control began.

This chain matters for preload. Continuous loading may shorten the interval from command to useful wrench, but the expected signature is a changed force-rise or stiffness state—not merely an earlier club release. Measuring several links allows mediation tests and prevents outcome timing from being mistaken for cause.

22.2.8 A Command-Shape Experiment

Compare an ideal step, a ramp, and a first-order activation filter with identical nominal onset and steady amplitude. The step supplies force immediately. The ramp and filter delay effective torque and can move the preferred nominal onset earlier. If a result exists only with an impossible step, it has limited human relevance.

Vary rise time across plausible bounds and optimize on training samples. Then freeze schedule or trigger for held-out cases. Report speed, face/path, peak load, and lower-tail outcomes. A robust timing claim survives command dynamics rather than hiding them.

22.2.9 Timing the Negative Couple

Define the couple interval by sign and swing-normal projection, then report its relation to state variables and impact. Test whether it consistently occurs over a state region across tempo changes. Compare direct torque, contact couple, shaft power, and relative release on aligned plots.

If the couple appears after the relevant state change, it cannot be its cause. If its onset precedes the change but varies too widely for prediction, the mechanism may be present without being a useful control target. Causal branches within the model and intervention data are still needed.

22.2.10 Timing Across the Two Allocation Extremes

The continuous arm-dominant strategy predicts continuity in one generalized drive or load mode across transition. The role-reversal strategy predicts a clearer allocation change, perhaps with continued co-contraction. Both may yield similar club event timing if activation and transmission histories differ.

Compare not just onset but duration, rise rate, impulse, and cumulative work. Align by transition, state trigger, and impact. A finding that only appears under one alignment should be described accordingly.

22.2.11 Robust Timing Is a Region

An optimum at 112 milliseconds before impact can be misleading if 108–125 milliseconds perform nearly as well while a narrow peak at 112 collapses under small errors. Map the objective surface and identify a plateau satisfying speed, control, and load constraints. Plateaus are more actionable than isolated maxima.

Uncertainty in event detection, sensor delay, and golfer variability should be shown on the same scale. If experimental timing uncertainty exceeds the width of a simulated optimum, the optimum is not currently testable at that precision.

22.2.12 Feedback Changes the Question

Open-loop schedules prescribe control by time. Humans use sensory feedback, feedforward preparation, and task-level corrections, although feedback delays limit rapid late changes. A state-triggered model approximates one aspect of feedback but is not a neural model.

Add bounded feedback controllers on later model rungs and ask whether the mechanism remains. A strategy that works only open loop may be fragile; a policy that relies on unrealistically instantaneous feedback is also suspect. Control assumptions belong beside mechanical ones.

22.2.13 Experimental Reporting

Publish synchronized raw and processed time bases, latency corrections, event definitions, and confidence intervals. Preregister primary windows and permit exploratory state alignments with clear labels. Provide code that reconstructs events from released signals. This lets reviewers decide whether a timing claim is mechanical, measurement-driven, or an artifact of normalization.

22.2.14 Reader Lab: Compare Three Alignments

Take repeated trials and align them by impact, by transition, and by a preregistered state event such as an arm–club-angle crossing. Plot direct torque, hand couple, shaft power, and club acceleration with uncertainty. A feature that sharpens only under state alignment may be organized by mechanics rather than absolute clock.

Next fit a timing policy on part of the data and freeze it. Evaluate fixed-clock, normalized-phase, and state-trigger versions on held-out participants. Include sensor and command delay. Compare distributional outcomes and trigger failures, not only means.

Finally shift event detection within its confidence interval. If the claimed ordering reverses, the evidence cannot support fine timing language. State that the events overlap within resolution.

22.2.15 Timing as Preparation Plus Opportunity

A useful late action requires preparation: the system must arrive with suitable geometry, velocities, load, and shaft state. It also requires opportunity: enough remaining time and motion for force or work to affect impact. Acting earlier can spoil geometry; acting later can miss the window.

This two-part view unifies delayed transfer and preload. Backswing and early downswing arrange state; later dynamics determine which control channels are effective. It explains why one timestamp cannot be detached from the trajectory that created it.

22.2.16 A Minimum Timing Report

State event definitions, time base, normalization, sampling rate, latency, filtering, and confidence intervals. Show raw-time and normalized-time plots, then one preregistered state alignment. Distinguish command, EMG, force, torque, motion, and outcome events. Report the full response surface around any optimum.

If the preferred region is narrower than measurement or execution uncertainty, say that the current experiment cannot resolve it. If a broad plateau survives held-out testing, report the plateau rather than a false-precision timestamp.

22.2.17 Timing and the Two-Hands Mechanism

The contact couple depends on both state and bilateral force. Its onset may be better described by a region of hand separation, club orientation, and force projection than by a single millisecond. The model can propose this region; instrumented data can test whether it recurs.

Continuous loading and role reversal make different predictions about the events preceding it. One predicts force-mode continuity; the other predicts a sign change with retained or recovered stiffness. Aligning only at impact can blur these transition signatures. The protocol should preserve both transition and state views.

22.2.18 What a Timing Cue Cannot Establish

A cue such as “wait” or “hold the angle” may change several variables and may be implemented differently by each golfer. Its success does not validate a modeled torque schedule unless the predicted mediator changes. Conversely, a mechanism can be real without a conscious cue.

The distinction protects both scientific timing and practical interpretation.

This keeps practical experiments open but bounded. Measure what the cue changed, compare it with the model, and avoid treating subjective intent as a control input known to the equations.

22.2.19 Chapter Checkpoint

Every timing claim should answer five questions: which event, measured how, in which frame or state, with what uncertainty, and related to which later outcome? It should also distinguish command from force and motion. If a state trigger is proposed, the observable proxy and held-out performance must be shown.

The key lesson is not that clock time is useless. Clock and state are two views. Agreement across them is strong; disagreement is information about tempo, preparation, or measurement. Publishing both prevents an exact timestamp from becoming a universal myth.

For the allocation comparison, transition is the critical bridge. Measure load, stiffness, activation, and wrench through it. The timing hypothesis succeeds only if those mediators explain the later difference.

22.3 Where the Picture Breaks

The oven has a small set of observable states. A golfer's relevant state includes uncertain internal loads and activation. A convenient club angle may correlate with, but not cause, the favorable mechanism.

Optimization can exploit unrealistic switches or measurement-perfect feedback. Rate limits, delays, noise, and held-out tests are required before interpreting a schedule biologically.

Go Deeper: Read the timing counterfactual framework, torque-program methods, and robust delayed-command study.

Chapter 23

Fast Once Is Not the Same as Robustly Fast

23.1 A Concrete Picture

A racing setup that sets one perfect qualifying lap may be difficult to drive in wind, tire wear, or traffic. Another setup is slightly slower at its best but retains performance across disturbances. Asking which is "faster" requires a distribution and a task, not one number.

A swing program can maximize nominal club speed while becoming sensitive to timing, strength, shaft stiffness, or face orientation. Robust analysis asks what happens across declared uncertainty and on cases not used to choose the program.

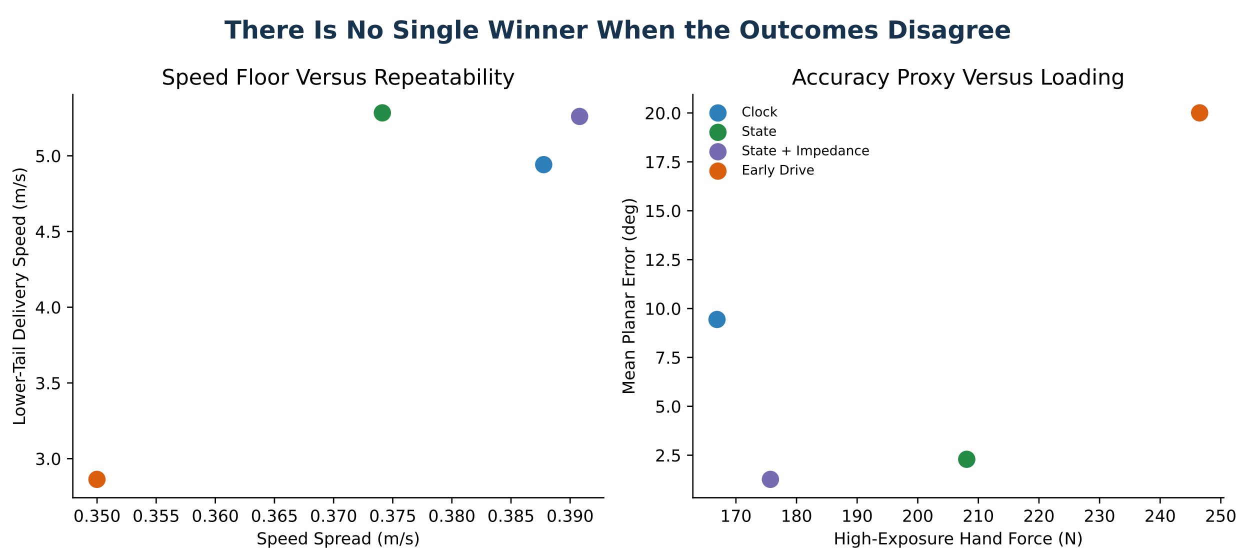

Figure 23.1: A Tradeoff Map Shows Several Nondominated Strategies Balancing Speed, Face-Path Proxy, Load, and Variability.

23.2 How the Mechanism Works

An ensemble samples uncertain parameters and conditions. Each control program produces an outcome distribution: median, lower tail, upper tail, failures, and secondary metrics. Programs are selected on training samples and evaluated on a separate held-out ensemble.

23.2.1 Lower Tail Versus Best Case

The fifth percentile asks how a strategy performs in difficult sampled cases. Improving it can matter more than raising the maximum. But the chosen percentile and distributions must be justified. Robustness is always robustness to something.

23.2.2 Pareto Fronts Preserve Tradeoffs

A strategy is dominated if another is at least as good on every outcome and better on one. Nondominated strategies form a Pareto set. Selecting among them requires values or context; physics alone cannot assign the weights.

In the coupled study, all eight preselected programs remain nondominated on held-out samples. Early opposing command improves lower-tail speed relative to late drive but worsens the planar face/path proxy. Declaring one globally best would erase the actual result.

23.2.3 Robustness Has Several Layers

Parameter robustness covers mass, inertia, stiffness, damping, delays, and torque capacity. Numerical robustness covers timestep and solver choices. Structural robustness asks whether the result survives another model tier. Participant robustness asks whether it generalizes to unseen people. These layers should not be blended into one confidence word.

23.2.4 Accuracy, Load, and Stability Matter

Clubhead speed is one outcome. Face, path, impact location, joint and contact load, actuator work, and constraint margin can oppose it. A strategy that uses high co-contraction may stabilize one coordinate while increasing effort. An allocation that minimizes hand force may increase joint torque. Reporting the surface is more useful than inventing a universal cost.

Model Result: The held-out ensemble rejects a universal optimum and exposes an explicit speed-versus-face/path tradeoff.

Human Evidence: Golf performance is participant-specific and variable; group-average kinematics cannot establish individual robustness (Glazier 2011; McPhee 2022).

Hypothesis: Preload or delayed transfer may improve robustness for some uncertainty families, but the benefit should be evaluated jointly with accuracy and load.

Practical Interpretation: Ask not only "how fast was the best swing?" but "how often, under which changes, with what face and load consequences?"

23.2.5 Maximum and Reliable Are Different Objectives

The fastest simulated trial is the maximum of one case. A robust strategy performs well across uncertainty in body parameters, club properties, timing, measurement, and execution. It may give up a small amount of nominal speed to avoid a large lower-tail failure. For sport, repeatable delivery and face/path control can matter more than a rare peak.

Write the objective before optimization. Possibilities include mean speed, fifth-percentile speed, probability of exceeding a threshold, face/path error, peak load, or a multiobjective combination. Weights encode values and should never be hidden inside a claim of “optimal.”

23.2.6 Build the Uncertainty Ensemble

Declare ranges and correlations for segment inertia, club properties, control amplitude and timing, rise time, grip geometry, shaft parameters, and initial state. Separate measured uncertainty from biological variability and deliberate design range where possible. Sample reproducibly and retain all failures.

An ensemble is not human evidence if its distributions are synthetic. It is a stress test of the model claim. Its value depends on transparency and on whether future measurements replace provisional ranges.

23.2.7 Train, Validate, and Test

Use one sample set to explore policies, another to select among them, and a final untouched set to estimate performance. If human data are used, split by participant. Repeatedly tuning after inspecting test outcomes turns the test set into training data.

Report baseline policies, not only the winner. Include simple constant or rule- based controls, the two allocation extremes, and ablations. A complex optimizer must earn improvement beyond these references and remain interpretable enough to falsify.

23.2.8 A Speed–Control Frontier

Plot terminal speed against face/path error, peak load, or sensitivity. A Pareto frontier contains solutions for which improving one objective worsens another. There may be no single best point. A user or experiment can select a region based on declared priorities without rewriting the mechanics.

For example, an early aggressive release might maximize nominal speed but have steep timing sensitivity. A later regulated release might lose 0.3 metres per second in the nominal case yet halve lower-tail face/path error. Whether that trade is attractive is a task decision, not a theorem.

23.2.9 Robustness of the Negative-Torque Strategy

Evaluate schedules with and without an opposing interval across the same ensemble. Compare sign persistence, terminal distribution, timing plateau, loads, and failure modes. If the negative interval helps only a narrow nominal case, describe it as a conditional mechanism rather than robust strategy.

The continuous-preload and role-reversal families should receive equal tuning budgets and constraints. Otherwise optimization quality can be mistaken for mechanical superiority. Cross-validate hyperparameters and publish unsuccessful regions as well as the selected frontier.

23.2.10 Sensitivity to Execution Error

Perturb torque amplitude, onset, duration, direction, and hand allocation around each policy. Use correlated noise and smooth command variations rather than only independent sample noise. Quantify outcome gradients and catastrophic boundaries.

A policy can be robust to onset error but fragile to direction or initial state. Radar summaries are convenient, but underlying response surfaces should remain available. Robustness is multidimensional.

23.2.11 Participant Variability

Different golfers can occupy different regions because of anatomy, strength, skill, club, and preferred coordination. A population policy may perform poorly for many individuals even when its average is good. Hierarchical models and participant-specific calibration can separate shared mechanism from individual control.

Personalization requires data. It should fail closed when parameters are unidentifiable or a participant lies outside training support. The product must not generate confident technique prescriptions from a generic synthetic model.

23.2.12 Validation Under Perturbation

Robustness claims are stronger when predicted responses to safe perturbations match measurements. Equipment property changes, constrained grip spacing, external timing cues, or small handle perturbations can probe sensitivity. The primary outcome should include recovery and variability, not only mean speed.

If humans compensate in ways absent from the model, that is evidence to add control freedom or narrow the claim. A model need not predict every adaptation to remain useful, but it must identify its validity region.

23.2.13 A Product That Communicates Risk

An open modeling product should return distributions, confidence, assumption flags, and out-of-domain warnings. It should show why a policy ranks well and which parameters can reverse it. Default views can be accessible, while downloadable tables and scripts support review.

This approach is deliberately unlike a recommendation engine selling one swing. It is a laboratory for asking how conclusions change. Its quality is measured by how clearly it exposes failure and uncertainty.

23.2.14 Reader Lab: Turn an Optimum Into a Distribution

Take the nominal fastest schedule and perturb initial state, inertia, shaft, command onset, rise time, amplitude, and direction. Plot terminal speed and control outcomes as distributions. Mark failure rate and lower tail. Compare with a slightly slower nominal schedule. The nominal winner may not be the reliable winner.

Build a Pareto plot and identify broad plateaus. Then hold out a new sample set or participants and evaluate without retuning. Show calibration: when the model says an outcome has 80 percent probability, does that event occur at roughly that rate? Poorly calibrated uncertainty should block confident product output.

Finally compare both allocation extremes under equal constraints and optimization budget. Include an intermediate family. A strategy can dominate in speed but be dominated in load or timing sensitivity. Preserve the frontier rather than compressing it into a single branded score.

23.2.15 Worst Case Is Not Automatically the Objective

Optimizing the absolute worst sampled case can be overly conservative and sensitive to arbitrary range edges. Mean optimization can ignore costly tails. Quantiles, conditional value at risk, and probability constraints offer middle grounds. Choice depends on task and consequence and should be exposed to users.

Robustness also differs between mechanism and policy. The negative-couple sign may be robust even if one command schedule is fragile. Report these separately so a weak controller does not prematurely reject sound mechanics—or strong nominal mechanics does not excuse a brittle policy.

23.2.16 Out-of-Domain Behavior

The model should detect when a participant, club, or state lies outside its qualified ranges. Extrapolation can change geometry, contact feasibility, or torque ranking. Returning a warning or no result is better than silently applying population averages.

Document which inputs define domain and which missing measurements prevent the check. An open tool can still invite exploration, but exploratory outputs should carry qualification labels that survive export into figures and PDF.

23.2.17 A Minimum Robustness Report

Publish the nominal case, uncertainty sources, distributions, correlations, sample convergence, failure handling, objective definitions, baseline policies, Pareto frontier, and untouched evaluation. Report mean, median, tails, and probability of satisfying all constraints. A single "robust score" is not enough.

Separate aleatory variability from epistemic uncertainty where possible. More trials may characterize execution variability; better measurements or models may reduce knowledge uncertainty. Combining them can be useful for risk but should not hide what future work can improve.

23.2.18 Robustness and Falsifiability

A broad claim predicts a distribution, not immunity to exceptions. Predeclare coverage and failure rates. If observed held-out outcomes fall outside intervals too often or failures cluster systematically, revise distributions or mechanism. Good uncertainty should itself be calibrated.

For the negative-torque and preload proposals, publish where the ranking reverses. Those boundaries guide safe experiments and prevent the product from presenting one strategy as universally superior.

23.2.19 Completion Criterion

The robust-control module is complete for a release when it can reproduce registered ensembles, compare equal-budget policy families, surface domain violations, and connect every summary to raw samples. It remains a model product until participant-held-out prediction meets declared gates.

23.2.20 Chapter Checkpoint

Before calling a strategy robust, name uncertainty, objective, acceptable failure rate, and qualified population. Show the nominal outcome, distribution, tails, tradeoff frontier, and held-out calibration. Preserve cases where the ranking reverses.

Robustness is not merely a statistical decoration on an optimum. It can change which policy is selected and what future measurement matters. It also provides the right boundary for practical use: a model that cannot characterize its failure region should not prescribe near that region.

For an open resource, this means the simulator must let users see assumptions and distributions rather than offering a single speed-maximizing button. A scientifically neutral product invites sensitivity analysis before celebration.

23.2.21 Comparing Robustness Across Model Rungs

A policy robust in a planar fixed-base model may become sensitive once base motion, shaft state, spatial contact, or measurement delay is added. Carry the same uncertainty definitions and outcomes upward. Separate uncertainty from richer physics from ordinary execution variability.

If a ranking changes, identify the added mechanism responsible. Shaft phase may narrow the timing plateau, or spatial hand geometry may reverse the couple in some states. These are useful discoveries. Show the cross-rung comparison rather than silently replacing the simple result.

Robustness also needs numerical qualification. Classify failed simulations, refine difficult cases, and ensure tail outcomes are not solver artifacts. A winner based on selectively missing hard cases is not robust.

23.2.22 Practical Interpretation of a Distribution

A broad plateau suggests several timings may work. A bimodal outcome suggests subgroups or competing mechanisms. A long failure tail warns against relying on the mean. An individual outside model support calls for measurement, not extrapolation. This language focuses on reliability rather than promising one secret motion.

23.3 Where the Picture Breaks

The race-car setup has measurable repeated laps. Model ensembles are constructed from assumed ranges and distributions. An omitted disturbance cannot be made robust by sampling known parameters more densely.

Pareto fronts also do not remove judgment. They display tradeoffs. Coaching, injury, and competition contexts would choose different regions and require human validation.

Go Deeper: Read the monograph's coupled uncertainty and control chapter and inspect the held-out evidence files.

Chapter 24

What Has Actually Been Measured in Golfers

24.1 A Concrete Picture

A courtroom separates what a witness saw from the explanation offered for it. Golf science needs the same separation. Researchers have measured segment motion, ground reaction, club motion, EMG, and some hand forces. Those observations constrain theories. They do not automatically prove the internal mechanism that produced them.

This chapter pauses the model story and asks what the human evidence really says, including sample sizes, methods, and gaps.

24.2 How the Mechanism Works

Kinematic studies reliably show that skilled swings involve coordinated segment motion and often proximal-to-distal peak timing. They can compare groups and outcomes. They cannot recover forces uniquely.

Inverse dynamics combines kinematics, external forces, and inertial estimates to calculate net joint moments and powers. It is sensitive to filtering, joint-center definitions, body-segment parameters, and contact allocation. A net wrist moment does not identify individual muscles or the separate hands.

24.2.1 Scapular and Wrist Activation

Jobe and colleagues measured scapular muscle EMG in 15 competitive male golfers, reporting phase- and side-dependent activity (PubMed record). This supports coordinated shoulder-girdle involvement. It does not establish hand-force direction or a unique scapular-to-club moment pathway.

Robinson and colleagues measured wrist-muscle activity in 15 subelite male golfers and found side/phase differences; downswing activation was not significantly related to the reported clubhead kinematics (PubMed record). This does not prove wrists are irrelevant. It shows the simple activation-to-outcome mapping was not established in that sample.

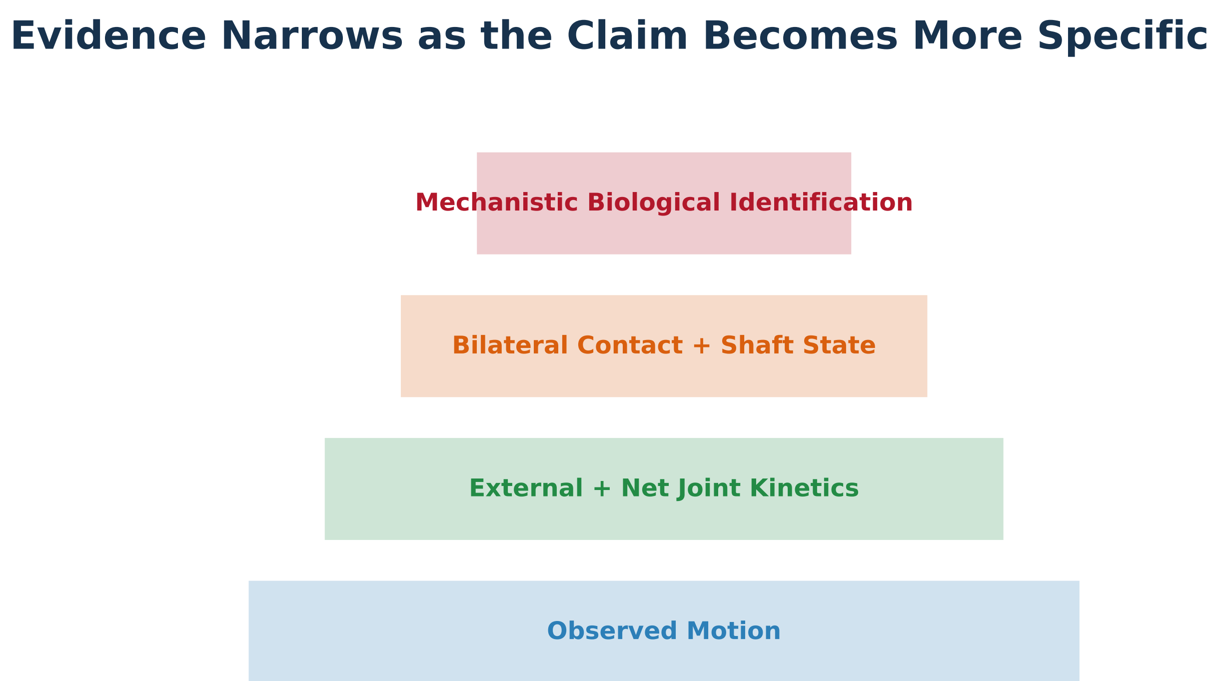

Figure 24.1: Evidence Rises From Observed Motion Through Net Kinetics and Contact Measurement Toward Mechanistic Biological Identification.

24.2.2 Instrumented Grip Evidence

Strain-gauge and six-axis grip systems demonstrate that bilateral contact can be measured (Koike 2016; Choi and Park 2020). Such measurements are critical because club motion and net wrench leave internal force modes unresolved. Sensor mass, calibration, coordinate alignment, and grip modification must be reported.

24.2.3 Heterogeneity Is a Result

A systematic review covering 92 golf-kinematics studies found methodological heterogeneity and contradictory findings (McPhee 2022). Differences in skill, club, task, filtering, phase definitions, and coordinate conventions limit pooled claims. This argues for open protocols and participant-level reporting rather than abandoning measurement.

24.2.4 What Is Still Missing

The proposed mechanism needs synchronized whole-body and club kinematics, separate ground reactions, bilateral six-axis hand wrenches and pressure, shaft strain or deformation, and forearm/shoulder/scapular activation. Calibration and residual gates must be fixed before outcomes. Participant-held-out validation is needed to test generality.

Human Evidence: Segment sequencing, phase-dependent activation, and measurable bilateral contact are established at bounded levels. The complete passive negative- couple and preload mechanism remains untested in humans. Professional-versus- amateur comparisons and biomechanics reviews add context without resolving that gap (Zheng et al. 2008; Hume et al. 2005).

Model Result: Simulations generate quantitative predictions for hand-force direction, couple sign, transmission gap, and timing that future measurements can refute.

Practical Interpretation: Treat human observations as constraints on the story, not decorative confirmation. When a required variable was not measured, say so.

24.2.5 A Ladder of Human Observations

At the most direct level are measured marker or image trajectories, force-plate signals, instrumented-grip wrenches, shaft strain, EMG voltage, and launch conditions. Each still has calibration and processing assumptions. Derived kinematics, inverse-dynamics loads, muscle-force estimates, and coaching interpretations occupy successively more model-dependent levels.

The evidence register should label every claim by this ladder. “Measured wrist torque” may actually mean a net moment estimated by inverse dynamics. “Scapular activation” may mean surface EMG from selected muscles, not scapular force or kinematics. Precision in wording prevents confidence from growing as results are retold.

24.2.6 Kinematic Sequence Evidence

Studies report proximal-to-distal patterns in many golfers, but segment definitions, skill levels, clubs, coordinate systems, and event methods vary. Peak order is descriptive and does not uniquely identify energy transfer. Three-dimensional component choices can alter the sequence.

Useful synthesis retains participant and method details. Instead of one universal curve, show distributions and contradictory findings. Link each assertion to the primary record and state whether data or only summary statistics are available.

24.2.7 Ground-Reaction Evidence

Force-plate studies show phase-dependent vertical, shear, center-of-pressure, and moment patterns and associations with speed or skill. Association does not establish a unique pathway to the club. Stance, task, foot allocation, filtering, and normalization affect results.

The proximal-distal framework can add value by predicting synchronized changes in ground, body, hand, and club observables. Existing ground evidence constrains plausibility but does not yet validate that complete chain.

24.2.8 Bilateral Grip Evidence

Instrumented clubs demonstrate that separate hand forces can be measured and that net club loading hides allocation. This is the most direct bridge to the two-hand wrench hypothesis. Yet small samples, sensor alteration, calibration, and limited synchronized measures constrain generalization.

The next dataset should include full six-axis wrenches or clearly declared components, effective contact locations, shaft response, club motion, and body kinematics. Releasing calibrated schemas and uncertainty is as important as publishing mean curves.

24.2.9 EMG Evidence

Scapular and wrist studies demonstrate phase- and side-dependent activation in their samples. EMG does not equal muscle force, and a lack of correlation with one club kinematic outcome does not imply the muscle is inactive or irrelevant. Crosstalk, normalization, electrode placement, contraction dynamics, and sample size matter.

EMG can test timing predictions and constrain biological allocations when combined with wrenches and motion. It cannot select a unique arm-versus-wrist strategy alone. High co-contraction may change stiffness and internal cost while net club wrench remains similar.

24.2.10 Shaft and Club Evidence

Strain gauges and high-speed tracking can measure deformation and presentation. Equipment properties affect the coupled system and may interact with individual delivery. A rigid-club model should be treated as a rung, not an assumption that shaft behavior is negligible.

Participant-specific club properties and calibration are required to compare modal predictions. A commercial stiffness label is not a sufficient parameter. Repeated trials should assess whether the predicted phase relation is stable.

24.2.11 What the Complete Dataset Is Missing

The central gap is not absence of all evidence. It is absence, within this project, of one synchronized, participant-held-out dataset containing bilateral grip wrench, whole-body and club motion, separate ground reactions, shaft state, EMG, and launch outcomes with a frozen protocol. Without it, links in the causal chain come from different samples and methods.

This is why the work labels simulation output as model result and human studies as bounded evidence. Combining them generates testable predictions; it does not convert them into validation.

24.2.12 Sample Size and Independence

Hundreds of swings from fifteen golfers are not hundreds of independent people. Trials nest within participant, session, and condition. Analyses need hierarchical structure and participant-level holdout. Sex, age, skill, injury, anthropometry, and equipment limit the population to which results apply.

Power calculations should target the primary mediator and effect, account for within-person correlation, and include attrition or sensor failure. Exploratory signals can guide later studies but should not be reported as confirmed after many uncorrected comparisons.

24.2.13 Data Quality Gates

Before hypothesis testing, require calibration pass, synchronization tolerance, force and motion range checks, coordinate registration, inverse-dynamics residual bounds, and complete provenance. Exclusions and missing-data rules are frozen. Identity-safe handling must not infer participant identity from row order or session filenames.

Publish deidentified derived data where consent permits, plus synthetic examples for pipeline tests. Synthetic records demonstrate computation, never human effect. A manifest should distinguish both unambiguously.

24.2.14 Evidence Can Disagree Productively

If measured hand couple has the opposite sign from the model, inspect frames and then revise the mechanism. If EMG timing differs while wrench prediction holds, the anatomical allocation—not necessarily the mechanics—may be wrong. If the model fits motion but fails ground and hand reactions, it is underconstrained or compensating.

Designing the evidence matrix this way prevents one failed layer from being hidden and helps target the next model or measurement. Falsifiability becomes a development tool.

24.2.15 How Readers Should Use the Literature

Follow live links to primary sources, inspect sample and methods, and distinguish what was measured from what was inferred. Systematic reviews help map heterogeneity but do not erase it. The open resource should retain a source register with stable identifiers, access dates, and claim-level links so future updates can revise conclusions transparently.

24.2.16 Reader Lab: Audit One Human Claim

Choose a sentence such as "elite golfers sequence segments proximally to distally." Follow its citation to sample, club, task, coordinate definition, filter, event method, and uncertainty. Ask whether every participant showed the pattern or only the mean. Then rewrite the sentence at the evidence level the study actually supports.

Do the same for “scapular muscles create the handle torque.” EMG may show phase- dependent activation, but without bilateral wrench and a closed-chain model the specific torque pathway is not identified. The revised sentence should preserve the activation observation and label the causal mapping as hypothesis.

This exercise is not rhetorical caution. It determines what new measurement is needed. The first claim needs standardized kinematics and kinetics; the second needs contact wrench, shoulder-girdle motion, activation, and uncertainty.

24.2.17 Evidence Integration Without Vote Counting

Ten studies reporting similar peaks do not automatically outweigh one study with different coordinates or direct force measurement. Compare design quality, measurement level, population, and relevance to the exact mechanism. A systematic review maps heterogeneity; a mechanistic model predicts why conditions might differ.

Where raw data are unavailable, retain the limitation. Digitized plots and summary statistics can inform priors but should not masquerade as participant- level validation. A living source register lets conclusions strengthen as open data and replications arrive.

24.2.18 Publication Table for Every Human Result

For each cited observation, report sample size at participant and trial levels, demographics, skill, club/task, measurement, inferred quantities, uncertainty, open-data status, and direct relevance to claim. Link the primary source. This table helps a phone reader grasp evidence without flattening its qualifications.

The modeling product can then color-code predictions as untested, partly constrained, replicated, or contradicted. Status must be derived from the evidence register, not manually styled in a marketing page.

24.2.19 A Minimum Human-Evidence Release

Publish the protocol, registration, consent-compatible data dictionary, calibration, processing code, exclusion flow, participant-level descriptive statistics, hierarchical analysis, and deviations. Link each result to its measurement and uncertainty. Keep raw protected authority separate when privacy requires it, and provide verification hashes or qualified derived data.

Model overlays should be predictions from a frozen version or clearly labeled fits. Show participant traces and residuals, not only grand means. A mechanism may hold for some people and fail for others; that variation is an outcome.

24.2.20 Updating the Framework With Data

Supportive human evidence can promote a prediction from untested to bounded observational or interventional support. It should not erase model assumptions or turn a sample into a universal technique. Contradictory evidence demotes or narrows the claim and triggers model review.

The lay book should update at the same time as the evidence register. A reader should not encounter an old confident explanation while the technical repository records a failed test. Publication controls are part of scientific integrity.

24.2.21 Why the Gap Is Worth Filling

The existing literature already supplies the pieces: motion, ground reaction, EMG, shaft behavior, and hand-force methods. A synchronized study is demanding but feasible. Its value lies in connecting those pieces within participants so the proposed causal chain faces one coherent test.

24.2.22 Chapter Checkpoint

Human evidence is already rich enough to constrain models and reject simplistic stories, but not yet combined here to validate the complete proposed pathway. The next step is connection, not selective citation: synchronized measurements, participant-level design, frozen predictions, and an evidence register that can record failure.

24.3 Where the Picture Breaks

The courtroom analogy implies observations are neutral. Motion capture and EMG are model-mediated measurements with artifacts. Surface EMG has crosstalk and amplitude limitations; markers move relative to bone; inverse dynamics uses assumed inertias.

Small specialized samples also limit population claims. Sex, age, skill, injury, and technique diversity require explicit sampling rather than extrapolation.

Go Deeper: Read the monograph's empirical evidence chapter, the model-and-measurement review (McPhee 2022), linked scapular and wrist EMG studies, and bilateral grip methods (Koike 2016; Choi and Park 2020).

Chapter 25

Muscles, Scapulae, Wrists, and Biological Ambiguity

25.1 A Concrete Picture

Several teams can move the same piano through a doorway. One team pushes from behind, another pulls with straps, and a third combines both. The piano follows nearly the same path, but internal effort, stability, and injury risk differ. Watching the piano does not identify the team strategy.

Club motion is the piano. Muscles, joints, two hands, and connective tissues form an overcomplete team. A model may estimate the net wrench required at the grip or the net moment at a joint. Those quantities rarely identify which muscles or scapular actions produced them.

25.2 How the Mechanism Works

Muscles pull through moment arms. Several muscles cross one joint; some cross two. Agonists and antagonists can be active together. A net joint moment is the signed sum of muscle, passive tissue, and contact effects. Infinite or numerous muscle- force combinations can reproduce the same net moment.

25.2.1 Activation Is Not Force

EMG reflects electrical activation near sampled muscle fibers. Muscle force also depends on length, velocity, tendon state, fatigue, architecture, and activation dynamics. Surface amplitude cannot be read directly as torque. Timing evidence is valuable when interpreted at that level.

25.2.2 Co-Contraction Changes Impedance

Equal opposing muscle moments can cancel in net torque while increasing joint stiffness and compression. This hidden activation may stabilize the grip or prepare rapid force transmission. It also costs energy and can raise load.

The arm-wrist comparison therefore needs effort and stiffness measurements, not only net club work. A model that minimizes generalized torque may choose an allocation unlike one that maximizes robustness or comfort.

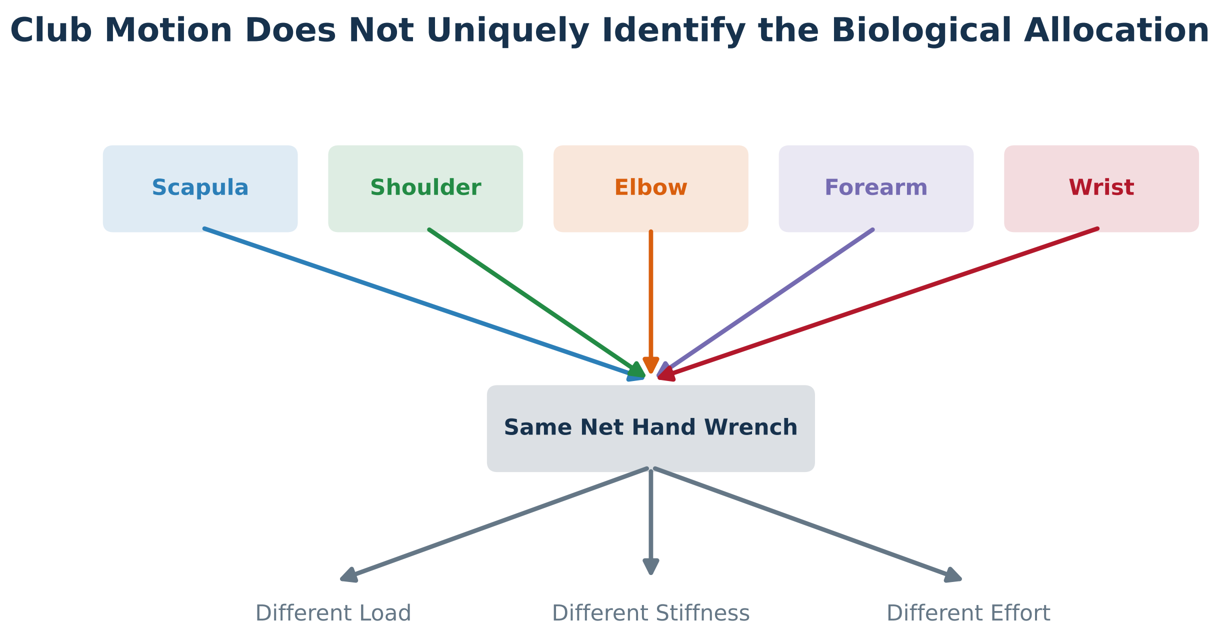

Figure 25.1: Many Muscle and Joint Allocations Converge on the Same Net Hand Wrench While Producing Different Internal Loads and Stiffness.

25.2.3 The Scapula Is Not a Fixed Hinge

Scapulothoracic motion reorients the shoulder socket and changes hand reach, moment arms, and muscle lengths. Retraction and protraction can contribute to a proximal push-pull pattern, but a reduced shoulder torque cannot be relabeled as scapular retraction. Direct scapular kinematics and muscle evidence are required.

Marker measurement over the scapula is difficult because skin moves relative to bone. Specialized clusters, imaging, or validated models may improve estimates. The uncertainty belongs in any mechanism test.

25.2.4 Wrists Are Multi-Axis Transmissions

Flexion-extension, radial-ulnar deviation, forearm rotation, carpal motion, and grip compliance interact. A planar wrist torque is a projection. Direct negative club torque could arise from active wrist moment, force-generated couple, passive stiffness, or a mixture.

25.2.5 Tendon and Muscle History

Tendon slack length is a model parameter, not a casual synonym for a loose swing. Series compliance delays and stores force. Muscle short-range stiffness depends on activation and recent movement. These phenomena may affect preload, but the dead-zone study represents only one falsifiable transmission family.

25.2.6 What a Responsible Biological Model Would Add

A higher-order model would include anatomical joint axes, muscle paths, activation and contraction dynamics, tendon compliance, passive limits, contact pressure, and participant parameters. It would be constrained by synchronized motion, wrench, EMG, and possibly ultrasound. Even then, muscle redundancy requires optimization or probabilistic inference and uncertainty reporting.

Human Evidence: Scapular and wrist EMG establishes phase-dependent activity in small samples, not a unique torque allocation (Jobe et al., 1995; Robinson et al., 2023).

Model Result: Generalized arm and wrist subspaces can produce the same club task with different internal demands. This is a mechanics statement, not muscle identification.

That many-to-one mapping is consistent with the broader multi-articular muscle coordination problem: muscle action depends on the state and on the rest of the mechanism, not on a muscle name alone (Zajac and Gordon 1989).

Hypothesis: Proximal push-pull and wrist resistance may maintain useful preload for some golfers, while wrist-led preparation and arm resistance may be favored for others. Bilateral force and stiffness data must decide.

Practical Interpretation: Similar club motion can be produced by different internal organizations. A visible outcome should not be reverse-engineered into one muscle story.

25.2.7 From Club Wrench Back Toward the Body

The club wrench is a useful boundary observation. Moving inward, each hand contact loads wrist and forearm; elbow and shoulder reactions depend on segment motion; the shoulder girdle transmits load to the trunk; trunk and pelvis couple to the ground. Inverse dynamics can estimate net generalized loads along this chain under a rigid-body model. It does not uniquely allocate those loads to muscles and passive tissues.

At each step, degrees of freedom and uncertainty multiply. Wrist axes are not perfect hinges. Forearm pronation changes orientation. The glenohumeral joint moves relative to a scapula that itself moves relative to the thorax. Skin markers imperfectly track bone. An anatomical story should become more qualified, not more certain, as it moves away from directly measured contact.

25.2.8 Muscle Redundancy

Many muscles can contribute to one joint moment, and muscles spanning several joints can move energy between segments. Agonists and antagonists may activate together, raising stiffness and internal force while leaving net moment nearly unchanged. Passive tissues can contribute state- and history-dependent loads.

An optimization criterion—minimum activation squared, metabolic cost, stress, or fatigue—selects one allocation from many. That solution is conditional on the criterion and muscle model. EMG can reject some allocations and inform timing, but sparse surface electrodes do not fully identify deep muscle forces.

25.2.9 Scapular Action as a Candidate Mechanism

Scapular protraction, retraction, rotation, and tilt reorient the shoulder socket and alter hand reach and leverage. Coordinated lead/trail shoulder-girdle action could plausibly contribute to a push–pull handle pattern. That makes scapular kinematics and activation relevant measurements.

It does not justify replacing a generalized arm driver with the phrase "scapular retraction." The same hand wrench can arise through different shoulder, elbow, wrist, trunk, and grip combinations. Direct or well-validated scapular tracking, multi-muscle EMG, and a closed-chain model are required to test the specific mapping.

25.2.10 Wrist Resistance Is Not Necessarily Passive

If net wrist torque opposes club rotation, active muscle, passive tissue, contact geometry, and dynamic interaction may all contribute. EMG silence does not prove absence of force, and EMG activity does not prove positive work. Shortening and lengthening contractions, co-contraction, and moment-arm changes complicate the relation.

The word "passive" should be reserved for a declared model channel or measured material behavior. A safer human hypothesis is that some late opposing club- moment contribution may arise without a newly increasing explicit distal drive. The experiment then measures wrench, activation, and state to test candidate sources.

25.2.11 A Many-to-One Example

Suppose the club requires a 12-newton-metre swing-normal moment and a specified net force. Allocation A uses a stronger bilateral differential hand force with modest contact moments. Allocation B uses smaller differential force plus larger wrist contact moments. Allocation C changes arm posture so common force also has a useful moment arm. The club can see nearly the same wrench in all three.

Internally, joint moments, co-contraction, tissue load, and sensitivity differ. Club motion alone cannot identify the allocation. Bilateral six-axis sensing rules out some solutions; EMG and kinematics constrain others; a model proposes remaining possibilities with uncertainty.

25.2.12 Preactivation and Effective Stiffness

Before motion reverses at transition, neural activation and load history may prepare muscles and tissues. Co-contraction can increase resistance to perturbation without a large net joint torque. Tendon and cross-bridge behavior can create history dependence. Grip and shaft deformation add mechanical states.

This gives a plausible biological basis for investigating continuous loading, but the simple dead-zone model is only phenomenological. Effective stiffness should be estimated through perturbation or system identification, not inferred from a feeling or one EMG channel. The benefit must also be weighed against internal load and reduced adaptability.

25.2.13 Lead and Trail Sides Need Separate Treatment

The hands occupy different positions and anatomical orientations. The same club-frame force direction maps to different joint and muscle demands on each side. Roles can change over the swing. Summing them into one "wrist torque" or one EMG average can erase the differential mode central to the hypothesis.

Report side, hand position, frame, phase, and dominance. A model may use symmetric contacts for mechanism isolation, but human validation should not assume symmetry. Participant handedness and technique further constrain generalization.

25.2.14 How to Compare the Allocation Extremes Biologically

Use matched club tasks and randomized, bounded strategy conditions. Measure bilateral wrenches, grip pressure, whole-arm and shoulder-girdle kinematics, forearm and scapular EMG, shaft response, and perturbation-based stiffness proxy. Track discomfort and peak loads. Verify that instructions actually changed the intended mediator.

Analyze whether force continuity or stiffness mediates outcomes rather than assigning success to the instruction label. Some participants may implement the same cue with opposite internal allocations. Cluster only on measured variables and validate clusters on held-out people.

25.2.15 Claims at Four Levels

Level one is observation: a measured hand force, EMG timing, or motion. Level two is rigid-body inference: net joint or interface load. Level three is biological inference: candidate muscle, stiffness, or tissue allocation. Level four is practical interpretation: a strategy might be worth testing. Every paragraph should make its level apparent.

Moving upward requires new evidence. A beautiful model trajectory cannot skip levels. This discipline makes room for bold hypotheses while keeping them easy to challenge.

25.2.16 What Would Refute the Anatomical Story

If predicted scapular kinematics or activation do not accompany the bilateral wrench, the specific scapular pathway weakens. If wrist activation and measured contact moments contradict the proposed passive resistance, revise the mapping. If multiple biological allocations remain equally supported, report ambiguity rather than choosing the most intuitive one.

The mechanical contact mechanism may survive even when an anatomical hypothesis fails. Conversely, observed activation may exist without the predicted club effect. Separating layers lets evidence refine the right part of the framework.

25.2.17 Reader Lab: Map a Claim Downward and Upward

Start with a measured negative hand couple. Moving downward toward raw evidence, identify sensor voltages, calibration, frame transform, hand positions, and uncertainty. Moving upward, identify inferred wrist moments, candidate muscle allocations, proposed scapular coordination, and practical interpretation. Mark where each new assumption enters.

Now ask which alternate biological allocation produces the same measured wrench. Construct at least two candidates and list observables that separate them: EMG timing, scapular kinematics, joint reaction, stiffness, or fatigue. The model should retain both until data discriminate.

Repeat from an EMG observation. Activation timing alone can support "this sampled muscle was active under this normalization." It cannot establish sign of club power. Linking both directions teaches why contact mechanics and biology need one another but cannot be collapsed.

25.2.18 Practical Value of Ambiguity

Redundancy can be beneficial: it permits adaptation to anatomy, fatigue, equipment, and perturbation. A modeling product should therefore expose feasible families and costs rather than force every golfer toward one allocation. The most stable individual solution may not resemble the population mean.

Ambiguity also sets a safety boundary. Without identified internal load, a speed-equivalent strategy cannot be assumed equally safe. Clinical and coaching decisions require measurements and expertise beyond the current model.

25.2.19 A Minimum Biological Claim

State the measured activation or kinematics, the net mechanical quantity, the model mapping between them, alternative allocations, and uncertainty. Do not name one muscle or scapular action as the source of a club wrench unless the data and model distinguish it. Report internal-load and safety limitations near any practical interpretation.

This minimum still permits useful conclusions: a candidate allocation is consistent with evidence; a proposed mapping is contradicted; or additional measurement is required. Scientific neutrality is not silence. It is calibrated specificity.

25.3 Where the Picture Breaks

The piano teams choose explicit roles. Human control can be distributed, automatic, and task-dependent. Muscle synergies and reflexes do not map neatly to coaching words such as push, pull, firm, or passive.

Musculoskeletal detail can also create false precision. Unmeasured moment arms or tendon properties may dominate an optimized allocation. More anatomy without better data can increase confidence faster than knowledge.

Go Deeper: Read the technical biological boundaries in the allocation chapter, the experimental protocol, and the linked activation evidence above.

Chapter 26

Design an Experiment That Can Say No

26.1 A Concrete Picture

A good smoke alarm is tested with smoke, with clean air, with a weak battery, and with a blocked sensor. A demonstration that rings only when the manufacturer presses a hidden button proves little. Scientific tests should likewise include conditions where the proposed mechanism predicts absence, reversal, or failure.

The proximal-distal hypotheses are valuable only if measurements can count against them. This chapter turns the model's attractive pictures into a protocol that can say no.

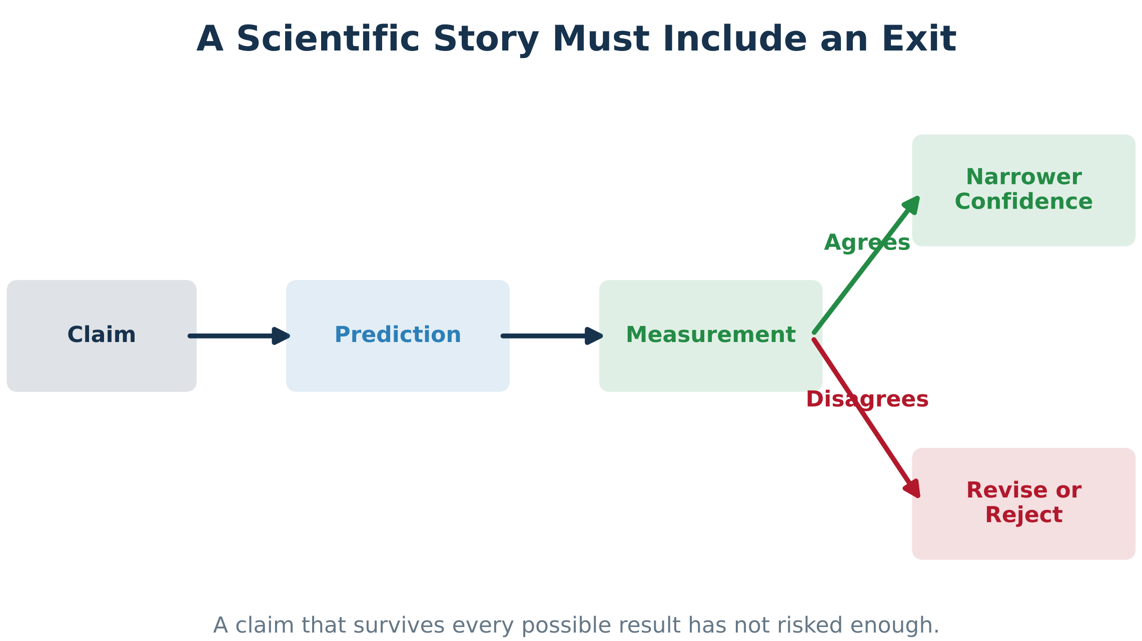

Figure 26.1: A Falsification Map Connects Each Mechanism Claim to Required Measurements, Negative Controls, and a Predeclared Failure Condition.

26.2 How the Mechanism Works

Preregistration freezes primary hypotheses, outcomes, windows, exclusions, filtering, coordinate conventions, and analyses before outcome inspection. It does not guarantee truth; it prevents the test from quietly changing to fit the data.

26.2.1 Synchronized Measurements

The protocol calls for whole-body and club kinematics, separate ground-reaction wrenches, bilateral six-axis grip wrenches and pressure, shaft strain or deformation, launch outcomes, and forearm/shoulder/scapular EMG. Shared timing is essential because millisecond phase differences are part of the mechanism.

26.2.2 Calibration Before Interpretation

Force sensors need zeroing, axis checks, and reference transforms. Motion and force coordinate systems must be registered. Club and body inertias need uncertainty. Known-load trials and action-reaction checks reveal sign or scale errors before a biological narrative is attached.

26.2.3 Filtering and Residual Gates

Differentiating marker positions amplifies noise. Filtering force and motion with incompatible cutoffs creates artificial phase shifts. The protocol freezes methods and checks inverse-dynamics residual force and moment. Trials failing closure are not used to support a mechanism.

26.2.4 Primary Falsifiers

The interaction-force claim is weakened if measured force direction, power, and counterfactual predictions do not match. The two-hand couple claim fails if bilateral wrenches lack the predicted late sign or geometry dependence. The preload claim fails if role reversal produces no measurable force/stiffness gap or if any advantage persists after gap-related variables are equalized. The robust-control claim fails if held-out participants do not preserve the declared tradeoff.

26.2.5 Negative Controls

Hardware or task manipulations should alter grip separation, shaft properties, or load direction within safe limits. Analysis controls scramble event timing, reverse sign conventions, or use impossible reference transforms to confirm the pipeline rejects them. Synthetic dry runs test code paths but never count as human outcomes.

26.2.6 Participant Holdout and Hierarchy

Repeated swings from one person are not independent people. Training and test splits occur at participant level. Hierarchical models report individual and population variation. A mechanism may be supported for a subgroup without becoming universal.

26.2.7 Identity and Governance

Human data require consent, privacy, controlled access, provenance, and an identity-safe analysis plan. Session or row order must not be used to infer player identity. Public releases can contain deidentified derived observables and qualified synthetic examples while raw authority remains governed.

Hypothesis: The predicted force-couple, preload, and timing signatures will survive preregistered participant-held-out testing.

Human Evidence: No such complete dataset has yet been analyzed by this project. The protocol and synthetic dry run establish readiness, not evidence.

Practical Interpretation: The strongest version of a claim names the result that would make its authors change their minds.

26.2.8 Turn Each Mechanism Into a Prediction Table

For every claim, list the model rung, intervention, primary observable, expected sign or ordering, uncertainty window, negative control, and failure condition. The table should distinguish direct measurement from derived quantity. This prevents a broad theory from retreating to a different outcome whenever one prediction fails.

For interaction-force transfer, predict hand-force direction, acceleration contribution, and forward persistence. For the negative couple, predict bilateral wrench projection and geometry response. For preload, predict force-rise or stiffness mediation. For robustness, predict held-out distributions rather than one optimum.

26.2.9 Phase One: Bench and Instrument Qualification

Before recruiting participants, calibrate force plates, grip sensors, pressure mapping, shaft strain, motion system, and timing hardware with known loads and motions. Characterize crosstalk, drift, hysteresis, range, orientation, latency, and repeatability. Verify action–reaction and wrench transforms on a rigid test fixture.

Run synthetic and hardware-in-the-loop signals through the complete pipeline. Intentionally swap axes, shift time, and exceed ranges to confirm gates fail. Qualification records belong in the release and should be tied to hardware and software versions.

26.2.10 Phase Two: Feasibility and Manipulation Checks

Use a small pilot to test comfort, sensor durability, event synchronization, and whether bounded instructions or equipment changes alter the intended mediator. Pilot outcomes do not confirm the primary hypothesis. They refine protocol and effect-size uncertainty before preregistration.

If an “arm-dominant” instruction does not change measured allocation, comparing its club speed with a “wrist-dominant” label is meaningless. Manipulation checks use bilateral wrench and activation/stiffness observables, not participant self-report alone.

26.2.11 Phase Three: Confirmatory Study

Freeze inclusion, exclusion, conditions, randomization, primary windows, outcomes, model versions, filters, transforms, residual gates, and statistical analysis. Power the study at participant level. Separate discovery and holdout participants and prevent identity leakage across splits.

Randomize condition order and allow familiarization. Blind data processing to condition where possible. Track adverse or uncomfortable trials. Preserve raw authority and generate deidentified derived releases under consent and governance.

26.2.12 Phase Four: External Replication

An independent group should receive the protocol, schemas, calibration fixtures or specifications, analysis container, and preregistered predictions. Replication with different hardware and participants tests whether findings belong to the mechanism rather than one laboratory pipeline.

Discrepancies should be decomposed by measurement, population, model, and intervention. A multi-lab result need not be identical to be informative; it should make heterogeneity explicit.

26.2.13 A Negative-Control Catalog

Geometric controls reverse hand positions or force directions and collapse separation. Temporal controls shift or scramble event alignment. Computational controls remove dead zone, shaft flexibility, or selected drivers. Sensor controls apply known loads and impossible transforms. Statistical controls test null data and label permutation.

Each control has an expected failure signature. If the pipeline still reports a negative couple after separation is mathematically zero, stop. If shuffled timing produces the same mediation result, timing evidence is not specific.

26.2.14 Primary Analysis for the Two Extremes

Compare continuous arm-dominant and role-reversed wrist-dominant families only after confirming measured allocation differs. Primary endpoints might include transition force-gap duration, effective stiffness, late swing-normal couple, club speed, and a face/path variability metric. Internal load and discomfort are safety outcomes.

Use hierarchical models with participant-specific effects. Test whether preload metrics mediate outcome differences. Report the full distribution and participants whose response reverses. Avoid declaring one universal strategy if heterogeneity is substantial.

26.2.15 Analysis of Slack

Predefine literal contact loss, near-zero differential load, force-development delay, and low stiffness as separate variables. Choose thresholds from sensor qualification or physiology, not outcome inspection. Report duration and phase, not only whether a gap occurred.

Test whether each variable predicts or mediates speed, control, and load after accounting for initial state. A finding that a brief low-load interval is common and beneficial would directly challenge a simplistic “never allow slack” claim.

26.2.16 Model-to-Data Comparison

Do not fit every parameter to each trial and then call the fitted trajectory a prediction. Estimate permissible calibration parameters on training trials, freeze them, and predict held-out wrench, strain, and motion time series. Use absolute errors, phase errors, sign accuracy, energy residuals, and coverage of prediction intervals.

Inspect residual structure. A consistent out-of-plane error suggests missing spatial dynamics; a timing-dependent shaft residual suggests missing modal physics; participant-specific force errors suggest allocation variability. Model revision follows a declared version and new holdout.

26.2.17 Decision Rules

Support is graded. A mechanism can pass geometry controls but fail human sign prediction; preload can alter stiffness but not performance; robustness can hold for a subgroup. Record each result separately. Do not collapse them into one "validated" label.

Predeclare what triggers rejection, narrowing, or redesign. A confidence interval crossing a practical threshold may mean inconclusive rather than no effect. Repeated confirmatory failure should demote the claim in the public evidence register.

26.2.18 Publication Regardless of Outcome

Register the study and commit to releasing qualified results whether supportive, null, or contradictory. Provide code, schemas, calibration, deviations, and limitations. Privacy-preserving data access can retain raw authority while sharing derived evidence and verification hashes.

An open resource earns trust when a negative result improves it. The model, figures, and lay explanation should update together so obsolete claims do not survive in an attractive PDF.

26.2.19 Reader Lab: Write the Failure Sentence First

Before analysis, complete this sentence for each primary claim: "We will narrow or reject the claim if _____ is observed under _____ qualified conditions." Include effect direction, uncertainty, mediator, and negative controls. Post it with the registration and analysis commit.

Then create synthetic data representing support, null effect, sign reversal, sensor failure, and participant heterogeneity. Run the frozen pipeline and check that it produces the intended decisions without crashing or silently excluding unfavorable cases. Synthetic success qualifies analysis logic, not the mechanism.

After real data arrive, publish the registered result and deviations before exploratory refinements. If the primary claim fails but a subgroup pattern appears, label it exploratory and design a new held-out test. This sequence keeps failure scientifically productive.

26.2.20 Choosing a Minimal First Human Study

The full synchronized protocol is ambitious. A first decisive study could focus on the bilateral negative-couple prediction with instrumented grip, club motion, and shaft strain, plus bounded grip-separation or geometry manipulation. It would not identify scapular strategy, but it could refute a central contact mechanism.

A second study could add transition stiffness and EMG to compare preload allocations. Staging reduces burden while preserving clear qualification. Each study should be designed around a discriminator rather than accumulating sensors without a primary question.

26.2.21 Stopping Rules and Governance

Define hardware-failure, discomfort, fatigue, and data-quality stopping rules. Record adverse events and protocol deviations. Protect participant identity and keep consent scope attached to every derived release. Scientific openness does not authorize disclosure of raw private data.

These controls belong inside the epic and evidence manifest, not in an appendix read only after collection. They make the proposed falsification program operational and responsible.

26.2.22 A Public Completion Checklist

Before calling an experiment ready, verify qualified hardware, frozen protocol, registered analysis, synthetic failure cases, participant-level power, identity and consent controls, calibrated time and frames, residual thresholds, adverse- event rules, and an outcome-independent publication plan. Each item links to an artifact and owner.

After collection, verify provenance, exclusions, code version, blinded primary result, deviations, uncertainty, and negative controls before interpretation. After publication, connect evidence status back to model and lay text. Completion is a closed trace, not only a paper submission.

26.2.23 If the Central Claim Fails

A failed negative-couple prediction would redirect attention to other contact, shaft, or control pathways. A failed preload mediation would remove justification for interpreting continuous loading through the dead-zone mechanism. A failed held-out policy would narrow optimization claims. None invalidates the general use of state, energy, and counterfactual analysis.

Writing these consequences in advance makes the theory genuinely revisable. It also reassures reviewers that the open resource is designed to learn rather than to sell a predetermined answer.

26.2.24 Chapter Checkpoint

The experimental program is credible when a reader can locate the registered prediction, calibration, data-quality gates, analysis, result, and consequence for the model. Every central mechanism has a feasible negative control and a result that changes its status.

This standard prevents “future work” from becoming an indefinite shield. Even a staged study can close one important uncertainty if it chooses a decisive observable and preserves qualification.

26.3 Where the Picture Breaks

The smoke alarm has a known target. Human strategies adapt to instrumentation and instructions. A manipulation intended to change preload may also change comfort, intent, or path. Manipulation checks and mediation analysis are needed.

Preregistration cannot rescue poor measurement or an underspecified model. Exploratory findings remain useful when labeled and independently replicated.

Go Deeper: Read the complete experimental falsification protocol, the model-falsification matrix, and the open qualification boundary.

Chapter 27

Open Evidence and the Reviewer Path

27.1 A Concrete Picture

A museum label can say a vase is ancient. A serious catalog adds provenance, material analysis, restoration history, photographs, and access rules. Reviewers can trace the claim instead of trusting presentation.

An open scientific resource needs the same layers. A polished PDF is the label, not the entire evidence. Source equations, scripts, parameters, generated data, figures, checksums, tests, and limitations let another person reconstruct the claim.

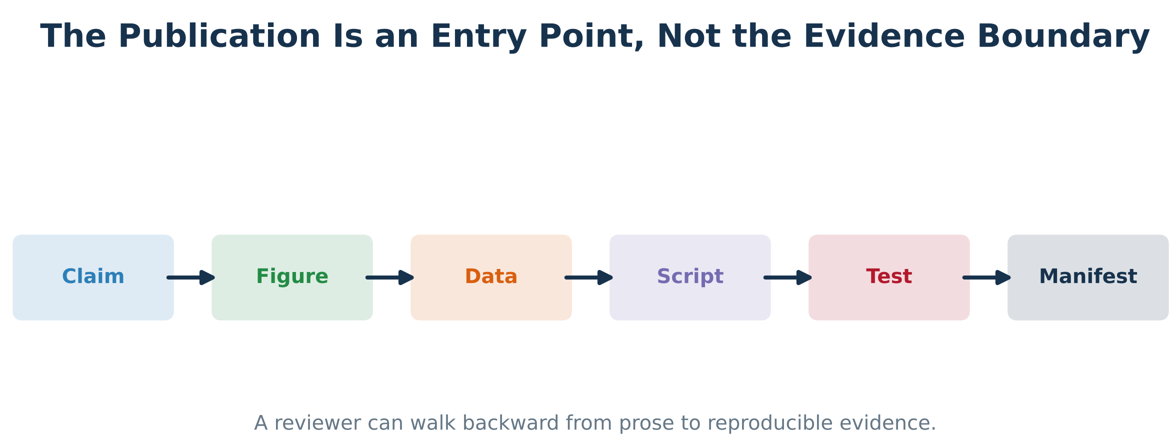

Figure 27.1: A Reviewer Path Runs From Claim to Figure, Data, Script, Test, Manifest, and Reproduction Result.

27.2 How the Mechanism Works

Each quantitative figure should point to machine-readable JSON or NPZ evidence. That evidence records parameters, conventions, software identity, outcomes, and residuals. A generating script rebuilds the figure. A test checks its invariants. A manifest lists hashes so reviewers know which artifact belongs to the release.

27.2.1 Determinism and Provenance

Random ensembles use fixed declared seeds and preserve sampled inputs. Build metadata records source commit and tool versions. Determinism does not mean the model is true; it means disagreements can be investigated rather than blamed on an unrecoverable run.

27.2.2 Live Links Are Part of Utility

This book links directly to technical source chapters, evidence directories, and cited literature. PDF URI annotations and HTML links are checked. A reader can move from a casual explanation to equations without searching a repository by filename.

27.2.3 Qualification Is More Than File Presence

The release validator checks hashes, required artifacts, PDF signature, link and outline counts, and generated-source freshness. Scientific tests check action- reaction, work-energy, constraints, limiting cases, counterfactual closure, and timestep behavior. Visual inspection catches clipped captions or unreadable plots that structural tests cannot.

27.2.4 A Reviewer Workbench

The reviewer path groups evidence by claim: interaction forces, hand-path work, two-hand couple, shaft, preload, spatial contact, uncertainty, and experiment. It states what each artifact supports and what remains unvalidated. Commands are copyable and fail closed on mismatches.

27.2.5 Open Does Not Mean Ungoverned

Code and synthetic evidence can be public. Human data may require controlled access. An open resource can publish schemas, derived deidentified results, analysis code, and provenance without exposing identities or licensed data.

Model Result: The current technical release is hash-bound and reproducible through a documented validator; paired figures and data expose the calculations.

Human Evidence: Release qualification does not add human evidence. It makes the absence and future path auditable.

Practical Interpretation: Trust should be able to travel backward from the sentence you read to the numbers, code, assumptions, and test that produced it.

27.2.6 The Reader Path Should Be Continuous

A casual reader begins with a figure and plain-language mechanism. A technical reader follows "Go Deeper" to equations and methods. A reviewer follows a figure to its source table, generator, model manifest, tests, and exact commit. These are not separate products. They are views of one evidence graph.

Links should target specific chapters, files, records, or DOI pages rather than generic repository roots. Stable anchors and release tags protect review from later navigation changes. The PDF must preserve clickable links and accessible bookmarks.

27.2.7 Canonical Source and Generated Artifacts

Text, data, figure scripts, and model configuration are canonical inputs. HTML, PDF, SVG, and rendered tables are generated outputs. Generated artifacts can be versioned for convenient review, but edits belong in canonical source. Build checks verify that public copies match source outputs byte for byte where that is the contract.

Every artifact should record or link to source commit, generation command, toolchain version, and content hash. A reviewer can then reproduce it or detect drift without trusting a filename such as "final."

27.2.8 Evidence Manifests

A machine-readable manifest lists claim ID, evidence level, model rung, input files, outputs, units, frames, qualification status, limitations, and citations. Unknown or provisional fields fail closed. Synthetic data are labeled so they cannot be mistaken for participant evidence.

The manuscript cites claim IDs or linked artifacts. When new human evidence arrives, updating the manifest and affected passages becomes traceable. This is more maintainable than burying qualification in a long disclaimer.

27.2.9 Reproducible Environments

Lock runtime and dependencies, record operating-system assumptions, and provide one command for each primary analysis. Tests cover physics contracts, schema, citation integrity, title convention, rendering, and public artifact identity. Random processes use recorded seeds.

Reproduction should not depend on a proprietary graphical interface. Optional GUIs and workbenches call canonical libraries. A reviewer can use command-line tools and open formats to inspect every scientific result.

27.2.10 Figure Provenance

Each figure has a descriptive caption and alt text, paired vector formats for web and print, and source data. The plotting script freezes units, signs, colors, and accessibility choices. Multi-panel figures label model result, human evidence, and hypothesis where applicable.

Screenshots may document a three-dimensional model or interactive view, but they should link to the run and camera settings. A screenshot is illustration unless its data and generation path are auditable.

27.2.11 Live References

Bibliography entries should contain DOI, PubMed, publisher, or stable repository links where available. Claims in the lay text link to primary sources and to the technical treatment. Automated checks verify citation keys and URLs, while periodic review handles redirects or changed access.

Links do not replace a bibliography. The rendered PDF should contain complete reference metadata so it remains useful offline, while hyperlinks improve reviewer utility on phones and computers.

27.2.12 Issue and Epic Traceability

Each chapter and cross-cutting requirement has a subissue with explicit acceptance criteria. Pull requests link the issues, explain evidence and limitations, and include local and CI verification. The parent epic summarizes completion only after merged artifacts are publicly reachable.

Issue count is not proof of scientific quality. It is a way to prevent thin coverage, preserve decisions, and let later contributors see why a chapter or control exists. Duplicate or obsolete work is closed transparently.

27.2.13 Open Review Without Software Promotion

The product should be described as an open research resource. Examples may use repository implementations, but conclusions are expressed in physical quantities and portable contracts. Independent implementations are welcomed and their disagreements retained.

Avoid framing every result as a benefit of one package. A reviewer should be able to reproduce equations in another tool and still participate. Scientific value comes from testable definitions, evidence, and criticism—not platform adoption.

27.2.14 Qualification Status

Each release states which models are numerically verified, which predictions are synthetic, which human studies provide bounded context, and which hypotheses remain untested. Unresolved publication pins, missing calibration, or failed gates block promotion to a stronger status.

Status should appear near the relevant claim and in a consolidated register. Readers should never need to infer qualification from repository history.

27.2.15 Long-Term Preservation

Release source and artifacts under appropriate open licenses, archive stable versions, and preserve metadata. Large PDFs and graphics need storage policies that support review without weakening integrity checks. Content hashes and release assets can handle files larger than ordinary source limits.

Migration plans should keep links resolvable. If an external source disappears, retain its citation metadata and a lawful archival pointer where available.

27.2.16 A Reviewer Exercise

Choose one figure. Follow its link to data, rerun the generator, compare hash, inspect the model manifest, and trace the claim to a test and source. Then alter one parameter or negative control. A healthy evidence product makes this path short and explains changed results.

The ability to disagree reproducibly is the central deliverable. A book that is easy to read and easy to challenge can serve both newcomers and specialists.

27.2.17 Reader Lab: Perform a Ten-Minute Trace

Open one chapter figure on a phone. Follow its caption link to the exact source data, inspect units and qualification, then open the generator and model manifest. The path should not require repository expertise. Back in the PDF, follow the primary citation and the deeper technical chapter. Record any dead end.

On a development machine, rerun the figure from the documented command. Compare output hash or numeric tolerance. Change one parameter and verify the artifact is visibly and semantically marked as exploratory. Then run the physics and publication gates.

If this path takes hours, the evidence may be open in principle but inaccessible in practice. Improving navigation, schemas, and small reproducible examples can advance review more than adding another framework layer.

27.2.18 Versioning a Living Scientific Book

Each release should state what changed in claims, evidence, model, and qualification. Stable chapter and claim identifiers let external reviews point to exact content. Corrections remain visible; old releases are archived rather than overwritten without trace.

Large artifacts can live in release assets or suitable large-file storage while source, manifests, and hashes remain in version control. The public URL should resolve without authentication and return the intended content type. A local PDF and deployed copy should be compared before completion is claimed.

27.2.19 Contribution Contract

Contributors can add a model rung, dataset, replication, or critique by satisfying shared schemas and tests. They need not adopt one engine or endorse the theory. New claims name falsifiers and evidence level; new figures include source and alt text; new human data retain consent and identity safeguards.

This contract makes disagreement composable. A contradictory implementation can sit beside baseline and change qualification through evidence rather than being treated as a software bug to hide.

27.2.20 A Minimum Release Bundle

The release should contain canonical chapter source, bibliography, figure data and generators, paired web/print vectors, PDF and HTML, model and run manifests, schemas, tests, exact commit, licenses, and qualification register. Public URLs are checked anonymously. The PDF is checked for bookmarks, live links, readable figures, and mobile-scale text.

Large artifacts should use an explicit storage route rather than an arbitrary small-file gate. Integrity comes from hashes, provenance, and reproducible generation, not from keeping scientific documents under one megabyte. Source review remains practical because generated binaries are separated from canonical edits and compared by contract.

27.2.21 Keeping the Resource Scientifically Neutral

Lead with questions, definitions, and evidence. Mention implementation only when it enables reproduction. Use generic physical terms in conclusions and link to portable schemas. Compare independent engines and preserve disagreement.

Avoid page labels that describe the production request, promotional claims, or repetition of repository branding. The reader should experience a scientific book backed by open resources, not a brochure about the resources themselves.

27.2.22 The Maintenance Loop

New issue proposes a falsifiable change. Tests define the contract. Canonical source or engine changes. Local numerical and visual gates run. Protected review and CI merge the change. Public artifacts are verified. Evidence status and chapter links update. Obsolete issues and outputs are reconciled.

This loop is deliberately mundane. It is how a large living document remains trustworthy after the initial authoring effort.

27.2.23 Chapter Checkpoint

Open evidence is not a folder of files. It is a navigable, versioned chain from claim to source, model, run, data, figure, test, limitation, and public artifact. The chain must work for a phone reader and an independent technical reviewer.

When a link breaks, a model fails, or new evidence contradicts a claim, status updates at every view. Protected review and reproducible builds preserve the history. This is how a large publication stays scientifically neutral after its first release.

The ultimate test is whether an independent person can reproduce a result, change an assumption, disagree, and contribute the disagreement without needing permission from one software implementation.

The repository should also publish a compact reviewer index. For every major claim it names the accessible explanation, technical derivation, model rung, data output, figure, tests, qualification, and open issue. A static PDF can link to this evolving index while release tags preserve the reviewed version.

Automated link checks cannot prove a source supports a claim. Human review still verifies relevance, study design, and wording. Automation catches broken routes, missing keys, stale artifacts, and inconsistent titles so reviewers can spend time on scientific judgment.

27.3 Where the Picture Breaks

The museum catalog can preserve a stable object. Software dependencies and web links change. Exact hashes improve traceability but long-term preservation also needs archived releases and documented environments.

Reproducibility can reproduce a systematic mistake. Independent formulation, adversarial review, and new data remain necessary.

Go Deeper: Start with the reviewer workbench, open release qualification, and data dictionary.

Chapter 28

A Practical Synthesis Without a Swing Prescription

28.1 A Concrete Picture

A weather map can explain pressure, wind, and uncertainty without telling every person whether to carry an umbrella. The decision depends on destination, tolerance for rain, and changing local conditions. A mechanics model can clarify the swing in the same way without declaring one motion correct for every golfer.

We can now assemble the picture: proximal work moves a linked system; constraints and inertia transmit and redirect effects; geometry changes leverage; two hands create visible and hidden wrench modes; negative torque can retain or stabilize; the shaft stores and returns energy; timing depends on state; robustness exposes tradeoffs; human evidence remains the final gate.

28.2 How the Mechanism Works

The useful framework is a sequence of questions.

First, choose the system and outcome. Are we explaining club kinetic energy, clubhead velocity, face/path, joint load, repeatability, or some combination?

Second, reconstruct the state and geometry. Where are the segments, how are they moving, and what elastic or activation states are loaded?

Third, close the instantaneous mechanics. Which forces, moments, constraints, gravity, and velocity terms balance acceleration? Are frames and reference points consistent?

Fourth, close work and energy over time. Which interfaces supply, transfer, store, return, or dissipate energy? Do positive and negative contributions cancel?

Fifth, intervene fairly. Hold state for pointwise attribution, integrate matched branches for persistence, and resimulate full schedules for strategy. Include negative controls.

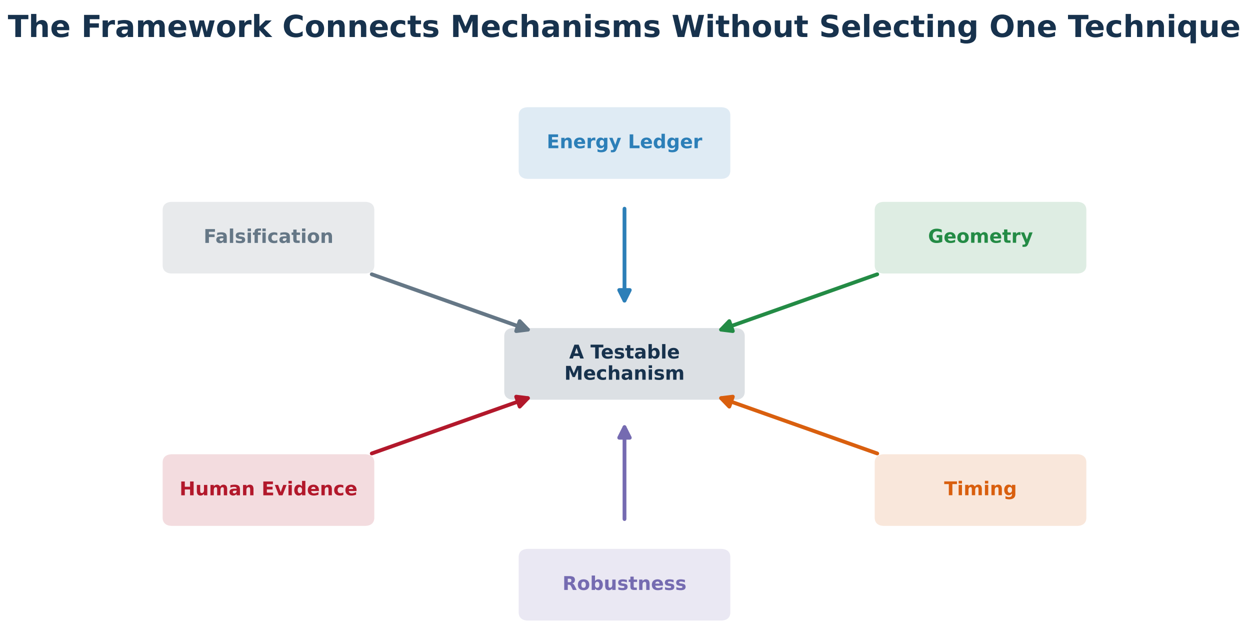

Figure 28.1: A Final Map Connects Energy Supply, Transfer, Storage, Geometry, Timing, Robustness, and Falsification Without Selecting One Technique.

Sixth, climb the model ladder. Does the mechanism survive two hands, compliance, a moving base, spatial motion, independent engines, and uncertainty?

Seventh, design the human test. Measure the variables that distinguish competing stories and preregister what would count against each.

28.2.1 What the Current Evidence Supports

The reduced models support a real mechanical possibility: club acceleration and hand-path work can be strongly influenced by interaction and state-dependent terms even with selected distal torque removed. Two-hand geometry can create a late negative club couple. Flexibility can modify timing while obeying energy closure. Arm- and wrist-dominant allocations can produce the same club task with different internal demands. Preload continuity helps only under a declared transmission family.

28.2.2 What Remains Open

The work does not establish a universal human technique, scapular mechanism, muscle preactivation strategy, injury benefit, or calibrated equipment effect. It does not identify one best allocation. The held-out model ensemble explicitly rejects a universal optimum across its outcomes.

28.2.3 Better Practical Questions

Instead of "should I hold lag?" ask which state variable and outcome the cue is supposed to change. Instead of "are the wrists passive?" ask whether direct wrist moment, impedance, and force-generated couple were measured separately. Instead of "is slack good?" ask which gap, stiffness, compliance, or activation variable was observed. Instead of "did the hips transfer speed?" ask which boundary power and energy balance supports that statement.

These questions do not make instruction impossible. They make claims testable and participant-specific.

28.2.4 A Reader's Next Experiment

Use the interactive workbench to compare clock and state triggers, remove a selected torque at one state, reverse a two-hand moment arm, or change shaft stiffness. Predict the result before running it. Then inspect forces, powers, work, energy closure, and falsifiers rather than only final speed.

Model Result: The project delivers a connected, openly reviewable set of mechanism tests across a common-observable ladder.

Human Evidence: The literature motivates and bounds the questions but does not yet validate the complete strategy mechanism.

Hypothesis: Some golfers may benefit from strategies that maintain useful proximal drive, manage distal resistance and preload, and permit state-dependent late transfer without force-transmission gaps. The opposite allocation remains a serious competing hypothesis.

Practical Interpretation: The framework is not a recipe. It is a way to ask better questions of a motion, a model, a sensor, or a coaching claim.

28.2.5 What the Framework Now Says

The swing is a coupled mechanical system. Proximal work, moving geometry, constraint reactions, direct distal control, bilateral hand allocation, shaft deformation, and ground interaction overlap. Clubhead speed is an outcome of their integrated trajectory. No single peak, torque sign, or body-part label contains the whole explanation.

The proximal-distal description remains useful when it means changing emphasis and energy pathways in a continuously linked system. It becomes misleading when turned into a strict relay or a universal timing script. The framework replaces that script with state, force, wrench, power, work, and counterfactual tests.

28.2.6 What the Models Establish

Verified reduced models demonstrate that interaction forces can carry and accelerate a distal link without direct distal drive; geometry controls the sign and leverage of that contribution; negative torque or contact couple can coexist with a positive later outcome; and multiple hand allocations can yield the same net club task. Flexible, moving-base, and spatial rungs test whether those mechanisms survive added physics.

These are model results under declared assumptions. Their strongest value is the observable predictions they generate. They do not establish a unique muscle strategy or prove that golfers exploit every simulated pathway.

28.2.7 The Two Strategy Extremes

One family emphasizes continuous arm/shoulder-girdle drive with wrist resistance from backswing through early downswing. Its proposed advantage is a loaded transition state with less transmission delay. The opposite family emphasizes greater wrist-created backswing club moment, arm resistance, then arm drive with wrist opposition in downswing. Its possible advantage is phase-specific use of different mechanical and biological resources.

Both can maintain contact and stiffness; both can create or avoid a measured force gap depending on implementation. Real strategies can lie between them. The meaningful comparison matches club task and measures bilateral wrench, stiffness, activation, work, load, and robustness.

28.2.8 A Decision Tree for the Researcher

If the question concerns linked inertia, begin with the double pendulum. If it concerns push–pull allocation, add two hands. If delay or "kick" is central, add shaft and transmission states. If ground or handle path is central, release the base. If out-of-plane couple matters, use spatial dynamics. If muscle claims are made, add biological measurements and a qualified musculoskeletal rung.

At each step, retain the smallest model that answers the question, common observables, negative controls, and closure tests. Add complexity only when a residual or hypothesis requires it.

28.2.9 A Decision Tree for the Evidence

If only motion is measured, report kinematics. With external wrenches and inertial assumptions, report net dynamics. With bilateral grip sensing, test hand allocation and contact couple. With shaft strain, test elastic pathways. With EMG and perturbation, constrain activation timing and effective stiffness. Do not climb the claim ladder without the corresponding evidence.

When evidence disagrees, localize the disagreement. A failed anatomical mapping does not automatically falsify the contact mechanics; a failed wrench prediction does. A model fitting speed but failing reactions needs revision.

28.2.10 What This Means for Creating Clubhead Speed

Within the mechanical framework, favorable conditions include sufficient whole- system work, a hand path and geometry that permit useful late club acceleration, control timing that avoids premature or incomplete release, and allocation that remains robust to execution error. An opposing moment can be useful if it shapes state and later power rather than simply wasting energy.

Those conditions are evaluation criteria, not body instructions. A specific golfer may meet them through different coordination. Speed must be considered with face/path, impact, internal load, and repeatability.

28.2.11 What "Manage the Negative Couple" Can Mean

Management is not necessarily conscious application of a negative wrist torque. It may mean arranging geometry and proximal motion so contact reactions have a favorable sign, maintaining enough engagement to avoid unwanted delay, and allowing direct control to regulate rather than fight the coupled motion. It may also mean avoiding excessive retention that leaves too little time.

The phrase should remain tied to measurable couple, power, timing, and outcomes. Without those, it is only a cue. The project does not recommend a cue from the current evidence.

28.2.12 A Practical Analysis Workflow

Begin with synchronized, qualified data. Reconstruct state and uncertainty. Close whole-system and interface ledgers. Plot force vectors and bilateral wrenches in declared frames. Run pointwise decompositions and matched-state forward branches. Compare model rungs and independent engines. Propagate parameter uncertainty and evaluate held-out participants.

Then classify each conclusion: mechanics identity, verified model result, human observation, hypothesis, or practical interpretation. Link it to the evidence graph. This workflow makes a sophisticated modeling product usable without pretending it is already a coaching authority.

28.2.13 Near-Term Completion Priorities

The highest-value next work is a qualified synchronized bilateral dataset, a spatial whole-body model with measured base and shaft state, participant-held- out prediction, and explicit arm/wrist allocation experiments. The workbench should expose counterfactuals, uncertainty, and failure regions with shared canonical engines. Documentation should update automatically from evidence manifests.

These priorities deepen falsifiability more than adding another attractive nominal animation. They also invite independent replication because the observables and contracts are software-neutral.

28.2.14 Questions a Reader Should Keep

What system and boundary are used? Which quantities were measured? Which were inferred or simulated? Does the ledger close? Is the comparison matched by state or task? Does geometry control the predicted sign? Can another allocation explain the same motion? Does the result survive uncertainty, higher model rungs, and held-out people? What finding would make the authors change their minds?

If those questions remain visible, the framework can grow without becoming a closed doctrine. It becomes an open map of mechanisms, evidence, and remaining uncertainty.

28.2.15 Final Boundary

Nothing in this synthesis establishes a universally superior swing, a prescribed scapular action, or a safe wrist-loading pattern. The research supports a more precise way to ask how transfer can occur and how competing explanations can be tested. Practical use requires qualified individual evidence and professional judgment appropriate to performance and health.

28.2.16 Reader Lab: Evaluate a New Claim

Suppose a new analysis says, “Trail-hand pull creates passive wrist lag and more speed.” Translate it into testable pieces. Define trail-hand force in a frame; define lag as state; define passive channel; identify power and work; specify the comparison and outcome. Ask whether arm, wrist, shaft, and geometry alternatives can reproduce it.

Then choose a model rung and negative controls. Predict bilateral wrench and shaft response. Match state for causal removal and club task for allocation comparison. Propagate uncertainty. Finally define a human measurement and a failure condition. Only after these steps should practical interpretation be considered.

This workflow does not drain intuition from the question. It converts intuition into an argument that another group can test.

28.2.17 The Desired End State of the Project

The complete product is a linked set of readable chapters, technical derivations, canonical simulation libraries, interactive experiments, qualified evidence, and reproducible publication. A newcomer can understand why a force vector matters; a specialist can audit equations; an experimentalist can see the next discriminating measurement; and an independent developer can implement the contracts elsewhere.

Completion does not mean the biological hypothesis is proven. It means the framework is sufficiently explicit, broad, and falsifiable that future data can change it cleanly. That is a more durable scientific objective than presenting one software package or swing strategy as finished truth.

28.3 Where the Picture Breaks

A synthesis map can create the illusion that every arrow has equal evidence. They do not. Structural mechanics, simulated model results, and human observations occupy different evidentiary levels. The claim labels must travel with any excerpt from this book.

The final danger is using scientific caution as vague neutrality. The models do make conditional predictions. They should be tested decisively, updated when wrong, and retained when they survive—not promoted prematurely and not dismissed because they are reduced.

Go Deeper: Read the monograph's discussion and conclusions, open the interactive workbench, and use the reviewer path to challenge any claim that matters to you.

Chapter 29

Walk Through a Whole Swing Without Losing the Ledger

29.1 A Concrete Picture

Imagine standing beside a complicated relay race. The runners do not merely pass a baton from one hand to the next. They are connected by springs, they can push on the track, several of them can hold the baton at once, and the finish line is moving. If we watch only the baton, the race can look beautifully simple: one runner speeds up, then another, and finally the baton crosses the line. If we want to explain *why* it crossed quickly, however, we need a ledger. Who did work? Which connection redirected a force? Where was energy stored? Which apparent contribution was just a change of coordinates? Which conclusion survives when we rerun the race with one action removed?

A golf swing is not literally that relay, but the picture is useful because it forces us to keep several questions separate. The clubhead can gain speed while the hands slow. A wrist moment can be negative while the clubhead continues to accelerate. A force can be large while doing little work because it is nearly perpendicular to the point velocity. A shaft can store energy during one interval and return it during another. Two hands can create the same net club wrench with very different internal loads. None of those statements is a contradiction. They become confusing only when the ledger columns are mixed.

This chapter follows one imagined swing from preparation to impact. It does not claim that this is the unique swing of a skilled player. Instead, it shows how to carry the book's concepts through one continuous story without quietly changing the system boundary, reference point, model, or meaning of a word.

29.2 How the Mechanism Works

29.2.1 Before Motion: Write Down the Accounting Rules

Our imagined observer begins before the club moves. The chosen system is the golfer plus club when asking about external work from the ground. It becomes the club alone when asking what the hands do to the club. It becomes the complete flexible club when asking about external grip work, but it may

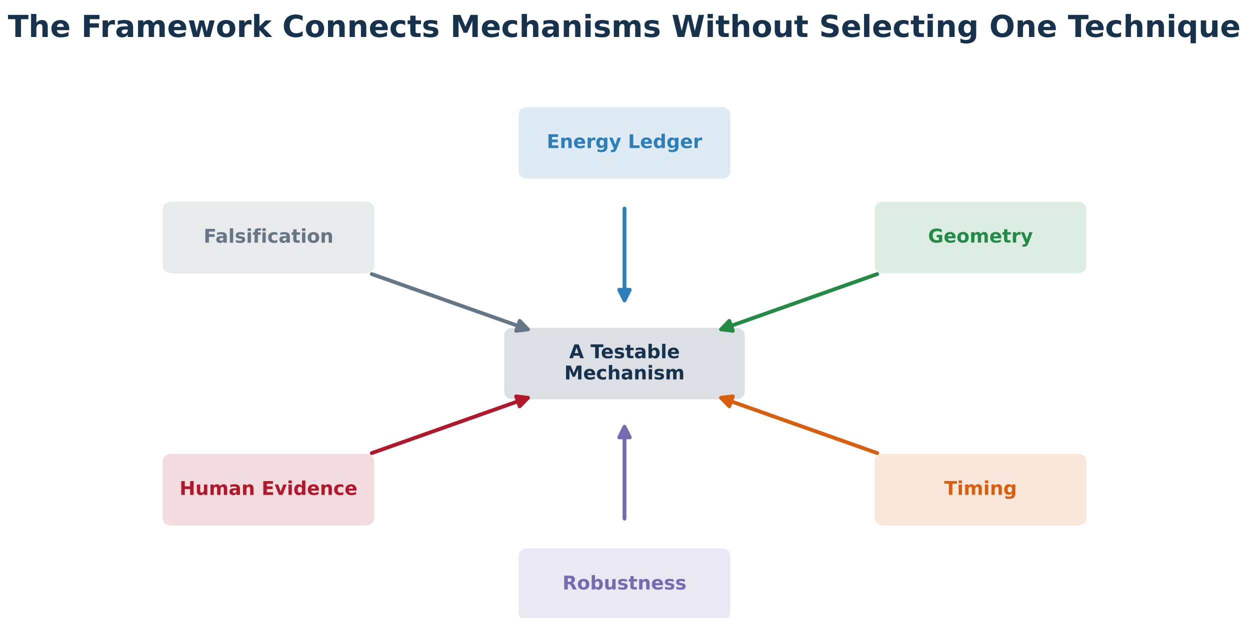

Figure 29.1: A Synthesis Map Connects State, Geometry, Forces, Power, Counterfactuals, Evidence, and Falsifiers Without Collapsing Them Into One Claim.

be divided into proximal and distal club segments when asking where shaft strain energy lives. Those are different ledgers. Changing between them is allowed; changing without saying so is not.

The observer also declares a laboratory frame, a club-attached frame, the grip center as the main wrench reference point, and positive angular directions. This sounds fussy until a force is transported from one hand to the grip center. The force remains the same physical interaction, but its reported moment changes with the reference point. Power remains invariant only when the wrench and velocity are transformed consistently. A persuasive diagram cannot repair inconsistent bookkeeping.

Finally, the observer lists the state variables available in the chosen model: segment angles and angular velocities for the simplest pendulum; pelvis and trunk motion for a moving-base model; shaft deflection and deflection rate for a flexible club; activation and series-force histories for a muscle-driven model. If a stored state is omitted, the same visible pose can hide different futures. The state list therefore determines which timing questions the model is capable of answering.

Practical Interpretation: Before asking which body part "caused" speed, ask what objects are inside the accounting boundary, what instant or interval is being studied, and which stored states are being carried forward.

29.2.2 Preparation: Motion Creates a State, Not Merely a Position

During the imagined preparation, the golfer moves the club away from the ball. The visible result is a backswing pose, but the mechanical result is richer. Segments have velocities, tissues and the shaft may carry strain, muscles have activation histories, the two hand contacts carry some distribution of force, and the ground has exchanged impulse with the body. Two golfers can arrive at nearly the same photograph with different values in those hidden columns.

This is why "the top" is not a complete initial condition. A model initialized from a still pose and zero velocity has erased the route into that pose. Such a model can still answer a deliberately posed question, but it cannot be used as evidence that preparation history is irrelevant. The continuous-preparation experiments in the technical work exist precisely to avoid resetting an internal transmission state at the command transition.

Consider the two allocation extremes developed earlier. In one, a proximal push–pull contribution remains directionally loaded while the wrists provide an opposing contribution. In the other, the modeled roles reverse across the transition. Both programs can produce the same net club-level moment before and after the change. They differ in the route taken by their internal states. If a declared transmission has a dead zone or finite force-development time, crossing through zero can create a temporary loss or delay of transmitted moment. If the transmission has no dead zone, that specific advantage largely disappears. The result is conditional on the element, not a universal verdict about wrists or scapulae.

29.2.3 Transition: Replace the Magic Instant With a Neighborhood

The swing reverses direction over an interval, not at a mathematical point that contains the whole explanation. Near transition, some segment angular velocities approach zero while accelerations and internal forces need not. The club may still be moving one way as proximal segments begin accelerating the other way. A graph aligned to a named event can make this overlap visible, but the event label does not create the mechanism.

Our observer therefore records a transition neighborhood. The ledger contains configuration, velocity, applied generalized moments, constraint reactions, hand wrenches, shaft state, ground-reaction wrench, and the power of each declared interaction. If the observer wants to discuss preactivation, activation must be measured or represented by a biological model. If the observer wants to discuss slack, the word is replaced by a quantity such as contact gap, tangent stiffness, low force, series strain, or force-development delay.

This disciplined language also makes opposing hypotheses stronger. Continuous loading might improve immediate transmission and perturbation resistance. A controlled unloading or role reversal might reduce internal loading, permit useful compliance, reshape the club path, or improve a participant's accuracy. The experiment must price both benefits and costs rather than declaring one side "connected" and the other "slack."

29.2.4 Early Downswing: The Ground and Moving Base Join the Story

As the downswing begins, the golfer pushes on the ground and the ground pushes back. The ground-reaction wrench is external to the golfer-plus-club system, but it is not a packet of energy that travels up the body like water through a pipe. It changes linear and angular momentum according to its impulse and does work according to the velocity of its application point and the chosen model. Some force components can be mechanically important by redirecting motion or satisfying constraints even when their instantaneous power is small.

Pelvis and trunk motion make the arm base move. In a prescribed-base pendulum, that motion is supplied as an input trajectory; the model does not pay the energetic or dynamic cost of producing it.

In a finite-mass moving-base model, the base responds to applied forces, restoring terms, damping, and connected segment reactions. The second model can test whether a mechanism survives when the base is allowed to react. It is not automatically more truthful in every parameter, but it closes a question that the prescribed model leaves open.

The familiar proximal-to-distal pattern may now appear in segment speeds. Pelvis speed peaks, then trunk, arms, and club. This is a useful observation, not yet a causal decomposition. Overlapping accelerations, interaction forces, joint moments, and changing geometry can produce the ordered peaks. A peak-speed sequence alone cannot tell us which term did work or whether deliberately slowing one segment would improve the final outcome.

Human Evidence: Measured golfers often display ordered segment-speed peaks, but timing varies with task, participant, club, event definition, and filtering. The observation motivates a mechanism test; it does not select one universal control strategy.

29.2.5 Mid-Downswing: Geometry Changes What the Same Force Can Do

The arms and club now form a changing geometric machine. A force at the hands has a moment about a joint according to its moment arm. The same force vector can create a large turning effect at one configuration and almost none at another. Likewise, the same joint moment can produce different accelerations because the mass matrix and dynamic coupling terms change with configuration.

Suppose a hand force points mostly along the instantaneous hand path. Its power at that point may be large because force and velocity align. Suppose another force is almost perpendicular to the path. Its direct power is smaller, yet it may redirect the path, maintain a grip constraint, or create a couple with the other hand. Calling the first force “useful” and the second “wasted” would miss the constraint and geometry columns of the ledger.

The double pendulum makes the changing leverage visible. When the arm–club angle is large, proximal motion can create a different induced acceleration of the club than when the segments nearly align. The model’s off-diagonal inertial terms are not a mysterious extra force; they express how coordinates in a linked system share kinetic energy and acceleration. A counterfactual can then ask how the club acceleration changes at the same state when one declared control moment is set to zero. That pointwise result is not the same as rerunning the rest of the swing without the moment.

29.2.6 Delivery: Two Hands Can Produce Force and Couple Simultaneously

Near delivery, reducing the hands to a single point hides an important degree of freedom. The resultant of the two hand forces translates the club. Their separation can also create a couple about the grip center. Direct wrist moments add another rotational contribution. Many combinations can yield the same net club wrench.

This is the setting in which a negative grip couple may coexist with increasing clubhead speed. “Negative” refers to the sign about a declared axis. The couple power is its moment multiplied by club angular velocity. Meanwhile, resultant force power, shaft energy return, gravity, and already acquired kinetic energy remain in the ledger. A negative couple can regulate release or reshape the trajectory without making the total club power negative. Conversely, a positive moment is not automatically beneficial if it acts at the wrong state or worsens impact geometry.

The two-hand planar model demonstrates a plausible passive late negative force-couple mechanism: after a coupled state has been created, the constraint forces needed to maintain both hand contacts can retain a negative couple even when a selected applied command is removed in a same-state or forward intervention. That is a model result about constraint dynamics. It does not show that the golfer's muscles are inactive, nor does it identify scapular retraction as the source. The body can remain active while a particular net club-couple component emerges from the linked dynamics.

Model Result: The executed two-hand models separate resultant force, the force-generated hand couple, direct wrist moment, and internal shaft moment. Their power and wrench identities close numerically under the declared model.

29.2.7 The Shaft: A Short Memory With Direction and Timing

The flexible shaft adds internal state. When it bends, it stores strain energy; when it straightens, it can return some of that energy. Damping dissipates a portion, and the deformation can alter clubhead orientation as well as speed. It is therefore misleading to describe the shaft only as a catapult that adds free energy. Its return comes from energy previously supplied to the club-body system, and its practical effect depends on timing, mode shape, damping, geometry, and impact conditions.

In our ledger, shaft strain energy is internal when the whole flexible club is the system. It becomes an exchange term when the club is divided into proximal and distal parts. Both descriptions are valid if the boundary is declared. The shaft can also make two visually similar delivery poses dynamically different because their bend and bend-rate states differ.

This gives "late release" a more careful meaning. A delayed visible angle does not, by itself, reveal whether the shaft is storing energy, whether wrists are doing positive or negative work, or whether a constraint reaction is creating the observed acceleration. Those mechanisms need their own measurements and model variables.

29.2.8 Impact: Do Not Let One Number Swallow the Task

Clubhead speed immediately before impact is important, but it is not the only task variable. Face orientation, path, strike location, dynamic loft, shaft state, and variability all affect the ball outcome. A simulated strategy that adds a small amount of peak speed while greatly increasing face or strike variability may not be better for the actual task.

The observer therefore reports a task vector rather than a victory number. That vector can include speed, path, face, strike, peak hand force, joint-load proxies, energy cost, and robustness to timing or parameter perturbations. The choice of weights is an objective, not a fact of mechanics. Different players and tasks can rationally occupy different points on the tradeoff surface.

At impact, the ledger also changes because the ball enters the interaction. The very short collision requires contact mechanics beyond the pre-impact swing model. The pre-impact state supplies initial conditions for that event; it does not by itself determine every launch outcome without a validated club– ball contact model.

29.2.9 Rewind the Swing With Counterfactual Scissors

Now the observer rewinds. First comes a pointwise decomposition: at selected states, set one declared control input to zero and recompute acceleration or constraint force without advancing time. This shows local sensitivity under the same state. Next comes a forward intervention: remove or alter the input and integrate, allowing the trajectory to diverge. This answers a different question about consequences over time.

The observer also runs negative controls. Coincident hand points should remove the force-generated couple. Consistent frame and reference-point changes should preserve power. Timestep refinement should not reverse the main result. Residuals should remain below declared thresholds. A mechanism that disappears under these controls is not rescued by a compelling animation.

Finally, the observer moves up the model ladder. Does the double-pendulum result survive a two-hand constrained model? Does it survive a moving base and flexible shaft? Does it survive spatial motion and parameter uncertainty? Do measured bilateral wrenches, kinematics, shaft strain, ground reactions, and activation timing show the predicted relationships in held-out participants? Each rung earns a narrower uncertainty reduction. None grants permission to skip the next rung.

29.2.10 Read the Finished Ledger in Plain Language

At the end, the observer can say something careful and still useful. The linked geometry allows proximal motion, interaction forces, hand-force couples, direct moments, and shaft state to influence distal motion in overlapping ways. Ordered speed peaks are compatible with transfer but do not uniquely identify it. Negative torque can coexist with a positive distal outcome because sign, power, system boundary, and time interval are different questions. Continuous loading and role reversal are separable hypotheses whose predicted differences depend on stiffness, gaps, activation history, and the task being optimized. The present models make those statements testable; they do not select a universal human technique.

That conclusion is less dramatic than “energy flowed down the chain,” but it is far more informative. It tells us which quantities to measure, which model states not to reset, which counterfactuals to run, and what evidence could make the explanation fail.

29.2.11 Reader Lab: Build a One-Page Ledger

Choose one interval—transition, mid-downswing, delivery, or impact approach. Draw six columns labeled state, external wrench, internal exchange, stored energy, task outcome, and falsifier. Fill each column with quantities rather than body-part stories. If you write “hips transferred energy,” replace it with a measurable term: pelvis-to-trunk joint power, interaction work over the interval, or a counterfactual change in club kinetic energy. If you write “wrists resisted,” specify joint moment, angular velocity, and the sign of their product. If you write “no slack,” name the gap, stiffness, force, or activation variable.

Then ask what your one-page ledger cannot establish. That final blank space is not a weakness. It is the beginning of the next experiment.

29.2.12 Chapter Checkpoint

The whole-swing story remains coherent only when the accounting rules stay visible. Preparation creates state. Transition is an interval. The ground changes momentum and can do work. Geometry changes leverage. Two hands create both resultant force and couple. The shaft stores and returns energy. Impact adds a new contact problem. Counterfactuals separate local contributions from forward consequences. Human evidence is required before a model allocation is called a biological strategy.

29.3 Where the Picture Breaks

The relay picture suggests discrete handoffs, whereas a real swing contains overlapping power, simultaneous constraints, distributed tissues, and feedback. The walkthrough also combines lessons from several model tiers; no single executed model currently contains every element in one participant-calibrated simulation. A coherent ledger is not proof that every entry is experimentally accurate. Parameters, soft-tissue motion, muscle forces, contact behavior, and individual objectives remain uncertain. The synthesis earns a research program, not a swing prescription.

Go Deeper: Compare the complete scientific monograph, the machine-readable transmission robustness evidence, and the reviewer workbench.

Chapter 30

A Curious Golfer and a Skeptical Reviewer Ask the Hard Questions

30.1 A Concrete Picture

Picture two readers at the same table. One is a curious golfer. This reader wants the mechanics to connect with sensations and visible motion: Why does the club seem to accelerate when the hands are not trying to fling it? Why might a negative torque help? Is "keeping tension" meaningful? The other is a skeptical reviewer. This reader keeps asking what was measured, what was assumed, which model produced the number, and what result would make the authors change their minds.

They are not opponents. The curious golfer prevents the explanation from hiding behind notation. The reviewer prevents the explanation from becoming a confident story that outruns the evidence. A mature open research resource needs both voices.

The conversation below is deliberately sustained. Each question is followed through mechanics, evidence, and limitations rather than answered with a slogan. The point is not to memorize a preferred response. It is to learn how to move from an intuitive question to a claim that can be checked.

30.2 How the Mechanism Works

30.2.1 "Is Proximal-to-Distal Transfer Just a Fancy Name for Sequencing?"

The golfer begins with the visible pattern: pelvis, trunk, arms, and club often reach peak speed in that order. The reviewer agrees that this is a useful kinematic description, then separates it from the mechanism. A sequence tells us *when* selected maxima occur. Transfer asks how forces, moments, constraints, work, and stored energy change the motion of a declared subsystem.

Two swings can show similar peak ordering while differing in joint power, ground-reaction impulse, hand forces, shaft deformation, and robustness. Two other swings can differ in peak timing yet reach similar club states at impact. The sequence is therefore one observable among many. It becomes mechanically explanatory only when connected to a model that predicts the measured forces and energy changes and survives counterfactual tests.

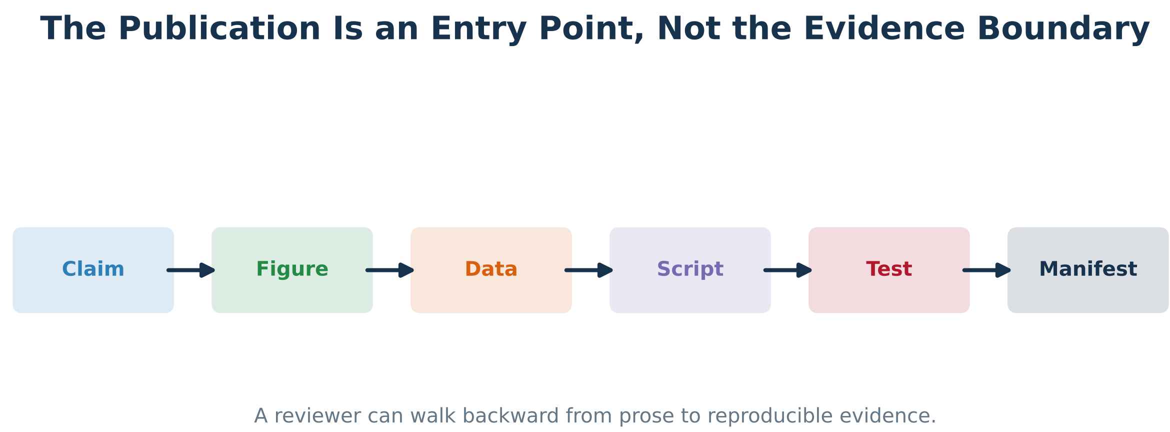

Figure 30.1: A Reviewer Path Moves From an Ordinary Claim Through Definitions, Model Tier, Observable Predictions, Negative Controls, and Evidence That Could Reject It.

The golfer's useful takeaway is not that sequencing is unimportant. It is that "good sequence" should be unpacked. Does it mean a reproducible ordering of segment speeds, a particular phase relation, favorable hand-path geometry, low peak loading, or robust impact conditions? Each meaning suggests different measurements.

30.2.2 "Does Energy Literally Travel From the Hips Into the Club?"

The everyday phrase sounds like a parcel moving through a delivery network. Mechanical energy is a scalar accounting quantity, not a material substance. When we isolate the club, hand forces and moments can do work on it. When we isolate the trunk, joint interactions and external forces appear at its boundary. With a flexible shaft, strain energy can be stored internally and later returned. The numerical transfers depend on where the system boundary is drawn.

The reviewer asks for a power balance. Force power is force dotted with point velocity. Couple power is moment dotted with angular velocity. Integrating power over a declared interval gives work. The sum must agree with changes in kinetic and potential energy after accounting for storage, dissipation, contact, and numerical residuals. If the balance does not close, the energy-flow story is incomplete.

The golfer can still use a simple image: energy changes hands through boundary work and internal storage. The image stays honest only if we remember that several exchanges overlap and that energy is not tagged with the name of the body part that first supplied it.

30.2.3 "If the Hands Slow Down, How Can the Clubhead Speed Up?"

Speed belongs to a point or segment, not to the whole chain at once. The hands can reduce their tangential speed while the club rotates faster about them. Changing geometry can convert the motion of the proximal endpoint into angular motion of the distal segment. Constraint forces and joint moments determine how that conversion unfolds.

A skater analogy helps but is incomplete. Pulling mass inward changes rotational inertia in a nearly isolated system; a golf swing is externally forced, linked, and actively controlled. The full explanation requires the arm–club geometry, mass matrix, applied moments, constraint reactions, and hand-path motion.

The reviewer therefore rejects the statement "slowing the hands causes clubhead speed" unless a counterfactual defines the intervention. Were the hands slowed by removing force, applying an opposing force, changing the path, or changing the entire upstream motion? Those interventions produce different futures. Observed correlation between hand deceleration and club acceleration does not select one of them.

30.2.4 "How Can Negative Torque Possibly Help?"

Torque sign is defined about an axis. Mechanical effect depends on the current state and on other terms in the equations. If the club angular velocity is positive and a club moment is negative, that moment has negative instantaneous power. Yet total club power may remain positive because resultant hand-force power, shaft return, gravity, or other moments exceed it. The club can also continue accelerating for a time because it already has momentum and because linked-system drift terms contribute to acceleration.

Negative torque may regulate release, alter face or path, manage shaft state, or reduce an overshoot. None of those possibilities makes negative torque automatically desirable. A negative moment applied too early, too strongly, or in the wrong configuration can reduce speed or disrupt the task.

The reviewer requests a signed plot of moment, angular velocity, moment power, resultant-force power, and net energy change. A plot of torque alone cannot support an energy claim. The golfer gains a better question: "What job is this negative moment doing during this interval, and what changes when it is removed in a matched counterfactual?"

30.2.5 "Is the Late Negative Couple Passive?"

The two-hand model shows that constraint forces can produce a negative couple after a coupled state has been established, including in an intervention where a selected command is removed. In that precise mathematical sense, continued application of that selected command is not necessary for the modeled couple over the tested interval.

That does not mean the golfer's body is passive. Muscles can remain active to maintain posture, hand contact, joint stability, and the state from which the constraint reaction emerges. Nor does it identify which muscles created the earlier state. "Passive mechanism" and "inactive person" are different claims.

The reviewer also demands the coincident-grip negative control. When the two hand application points are collapsed to one point, the force-generated couple should disappear. It does so in the executed planar model. That result supports the geometry of the couple; it does not validate the complete anatomy of a human swing.

30.2.6 "Do the Scapulae Create the Push–Pull Couple?"

Scapular protraction, retraction, rotation, and tilt change the shoulder socket and the geometry from which the arms act. They are plausible contributors to a bilateral push–pull pattern. Electromyography shows phase-dependent activity in selected shoulder-girdle muscles. Those observations make the proposal worth testing.

They do not make the mapping unique. The same net club wrench can result from different combinations of trunk motion, scapular motion, shoulder and elbow moments, wrist moments, grip pressure, and muscle co-contraction. Surface EMG reports electrical activation timing imperfectly; it does not directly report muscle force, hand-force direction, joint work, or a unique causal pathway.

The reviewer's minimum evidence package includes bilateral grip wrenches, full-body and scapular kinematics of adequate quality, closed-chain inverse dynamics constrained by the internal wrench, forearm and shoulder-girdle EMG, and participant-level replication. Until then, "scapular action is a candidate mechanism" is supported more strongly than "scapular retraction creates the club couple."

30.2.7 "If the Wrists Resist, Are They Passive?"

No. A joint can show little visible angular motion while muscles actively co-contract, while passive tissues bear load, or while opposing moments nearly cancel. A negative wrist moment can involve active eccentric work, isometric activation at small angular velocity, elastic tissue loading, or a combination. The word "resist" describes a net mechanical relation only after the axis, moment, motion, and interval are defined.

This distinction matters for the two allocation extremes. The proximal-drive case in a reduced model is not equivalent to "arms active, wrists off." The wrist contribution can remain negative and actively regulated. The wrist-led preparation case is not equivalent to "arms doing nothing." The arms must still maintain geometry and contact. Biological redundancy prevents net joint moments from revealing a unique activation pattern.

The golfer may feel firmness, pressure, or stretch. Those sensations are useful reports, but the reviewer will not equate them directly with stiffness or torque without calibration. A sensation can motivate an experimental condition; it cannot substitute for the measurement.

30.2.8 "Is Slack Bad?"

The reviewer first removes the word from the question. Does slack mean a literal loss of grip contact, a low-force interval, tendon slack length, low tangent stiffness, a transmission dead zone, delayed activation, or simply a relaxed sensation? These quantities behave differently.

A contact gap can delay force transmission and impair precision. Some compliance can store energy, filter impact, widen a controllable region, and protect tissue. Low co-contraction can reduce energetic cost and permit fluid reorientation; higher co-contraction can increase task-relevant stiffness and disturbance resistance at a cost. A tendon has a slack-length parameter without implying that the entire movement feels loose.

The executed dead-zone model makes one narrow prediction: when both actuator channels reverse through a declared gap, transmitted torque is delayed relative to a program that preserves loading direction. When the dead zone is removed, that advantage largely disappears. This is evidence about a model family, not proof that maximum tension is best. The next experiment should estimate the actual gap, stiffness, or force-rise behavior instead of arguing over the word.

30.2.9 "Which Is Better: Arm-Led Preparation or Wrist-Led Preparation?"

The honest answer is that the current evidence does not establish a universal winner. In the matched-task model, proximal-dominant and wrist-dominant controls can produce the same club-level moment with different hand-force and generalized- torque demands. The allocation that minimizes one demand changes with geometry and need not minimize another.

Persistent proximal drive with wrist resistance may preserve a loaded transmission state through transition. Wrist-led preparation followed by role reversal may exploit compliance, reduce a proximal load, or suit another coordination objective. Intermediate allocations may dominate both extremes for a particular participant. Speed, accuracy, tissue loading, repeatability, and adaptability are not the same objective.

The reviewer asks for a preregistered tradeoff surface and held-out participants, not a group-average declaration. The golfer is invited to understand the options, not to treat the chapter as medical or coaching advice.

Hypothesis: Allocation strategies will differ most when the transmission has measurable state dependence and when the task includes perturbation or accuracy constraints, not merely an unperturbed peak-speed target.

30.2.10 "Does Preactivation Eliminate Slack?"

Preactivation can increase force readiness and effective stiffness, but it does not erase every kind of slack. It may take up a contact gap or develop series force before a rapid command. It may also increase internal load, energetic cost, and sensitivity to unwanted high-frequency disturbances. Its effect is directional and task dependent.

The reduced biological bridge carries excitation, activation, muscle force, and series-force state continuously through a preparation-to-delivery change. It shows that two programs with identical net targets can retain different histories. The numerical difference in that simplified model is small, which is itself informative: the biological state makes route history observable, but it does not prove a large performance advantage.

The reviewer would measure pretransition activation, grip force, force-rise delay, stiffness proxies, and posttransition task error together. Activation without force and stiffness data is not enough. The golfer gets a balanced interpretation: readiness may matter, but "more activation everywhere" is not a scientifically supported prescription.

30.2.11 "Why Not Build One Huge Human Model and Settle It?"

A large model can represent more anatomy while introducing more parameters, uncertainty, and opportunities for compensation. If muscle paths, activation dynamics, contact properties, and participant scaling are poorly identified, the model may fit observed motion through many incompatible internal stories. Size is not the same as explanatory power.

The model ladder assigns each rung a question. A double pendulum exposes coupling and geometry. A two-hand constrained model exposes resultant and differential contact modes. A moving base tests whether prescribed support hid the mechanism. A flexible shaft carries strain state. A spatial model checks out-of-plane effects and frame conventions. A musculoskeletal model tests candidate biological allocations. Cross-engine agreement catches implementation errors but does not convert assumptions into measurements.

The reviewer prefers the smallest model capable of failing the claim being tested, followed by a declared fidelity gate. The golfer benefits because each result remains interpretable rather than disappearing inside a black box.

30.2.12 "Can Inverse Dynamics Tell Us Who Did the Work?"

Inverse dynamics estimates net generalized moments under a chosen model and measured kinematics. It is powerful, but a net joint moment can result from many muscle-force combinations. In a closed two-hand chain, unmeasured internal grip forces add another ambiguity. Filtering, inertial parameters, joint-center definitions, and soft-tissue artifact affect the result.

Joint power—net moment multiplied by joint angular velocity—describes net mechanical power at that modeled coordinate. It is not identical to metabolic effort, individual muscle work, or energy that can be tagged and followed into one distal segment. Biarticular muscles and constraints can redistribute power in ways that a simple local story misses.

The reviewer asks whether the internal wrench was measured, whether residuals close, how uncertainty was propagated, and whether alternative models change the conclusion. The golfer can still learn from joint-power curves, provided they are read as model-dependent net quantities rather than photographs of individual muscle action.

30.2.13 "What Would Actually Prove the Proposed Mechanism Wrong?"

This is the question that turns an explanation into a scientific program. The persistent-loading claim loses support if matched strategies show no predicted difference in force continuity, stiffness, delay, or robustness. The two-hand negative-couple mechanism fails at its model tier if the sign vanishes under timestep refinement, power identities do not close, coincident hand points retain a couple, or the result depends on a hidden prescribed motion.

The scapular pathway weakens if predicted scapular kinematics and activation do not accompany measured bilateral wrench changes, or if another allocation explains the data equally well. A universal sequencing claim fails if held-out participants achieve equal or better task outcomes through different timing. A shaft-energy claim fails if measured strain and energy return do not match the predicted phase and magnitude.

Negative results should be published with the same care as positive ones. A failed mechanism can leave useful invariants, validated measurement tools, and better-bounded alternatives. The reviewer is not asking for certainty; the reviewer is asking the authors to name the observations that would force a revision.

30.2.14 "Why Make the Evidence Open?"

An open paper is more useful when its figures, data summaries, scripts, event definitions, coordinate conventions, and residual gates can be inspected. Reviewers should be able to rerun a plot, change a parameter, and see whether a claim survives. Machine-readable manifests and hashes help distinguish a specific evidence snapshot from a later artifact.

Openness does not remove every barrier. Specialized engines, proprietary measurements, participant privacy, and computational cost can limit complete reproduction. The responsible response is to document those boundaries and publish the smallest sufficient evidence rather than implying that a static figure is the whole experiment.

For the golfer, open evidence means the explanation can improve without loyalty to a product or personality. For the reviewer, it means criticism can target a declared calculation instead of guessing how a result was made.

30.2.15 "What Can I Safely Take From This as a Golfer?"

You can take a way of asking questions. A swing creates a state, not merely a pose. Geometry changes what forces and moments can do. Two hands create both a resultant force and a couple. Negative torque is not automatically harmful. Compliance can help or hurt depending on the task. Peak sequence, joint power, EMG, and clubhead speed answer different questions.

You should not take a universal instruction to pull with one scapula, push with one arm, hold a wrist angle, maximize tension, or deliberately apply negative torque. The models have not established the safety or superiority of those instructions for an individual. Technique changes that load wrists, elbows, shoulders, or the spine belong with qualified coaching and clinical judgment.

The deepest practical lesson is that a good strategy must be judged as a whole: speed, face and path control, strike, robustness, effort, and load. A mechanism that sounds elegant in isolation may be a poor trade for a particular person.

Practical Interpretation: Use the framework to make coaching claims more specific and testable. Do not turn a model variable directly into a body cue.

30.2.16 Reader Lab: Hold Both Voices at Once

Choose a familiar claim such as "keep the club loaded," "fire the hips," or "let the club release itself." Let the curious golfer explain what the phrase feels like and what visible outcome it tries to produce. Then let the reviewer rewrite it using a state, force, moment, power, stiffness, or task variable. Name the model tier that could test it. Name one negative control. Name one human measurement. Finish with a sentence beginning, "I would change my mind if …"

If the scientific rewrite no longer resembles the original claim, that is valuable information. The phrase may have been a useful personal cue without being a literal mechanism. Cues and mechanisms can coexist, but they should not borrow each other's evidence.

30.2.17 A Full Conversation From First Impression to Testable Claim

The framework becomes easier to use when we slow down and follow one claim all the way through it. Imagine that a golfer watches a slow-motion swing and says, "The arms stopped, so their energy must have gone into the club." That sentence feels persuasive because two visible events occur together: the hands appear to slow and the clubhead appears to speed up. It is a reasonable observation. It is not yet a complete mechanical explanation.

The curious golfer should begin by saying what was actually seen. Which point on the hands was tracked? Was "speed" a linear speed in the laboratory frame, an angular speed about the shoulder, or an angular speed relative to the club? Did the apparent slowing happen before, during, or after the clubhead speed-up? How much did the club's direction change? A camera view can make a point look slow because it moves partly toward the camera. Even excellent motion capture needs a coordinate convention before "slowing" has one stable meaning.

Now the skeptical reviewer asks what "their energy" means. The arms have translational kinetic energy, rotational kinetic energy, gravitational potential energy, elastic energy in tissues, and interactions with the torso, hands, and club. Those accounts are not little colored parcels that can be watched crossing a joint. A segment can lose kinetic energy while muscles add positive work, because an even larger amount leaves through joint reactions. It can also gain kinetic energy while its net joint moment does negative power, because other links and constraints contribute more. The correct ledger needs the declared system boundary and every power crossing it.

For a first model, use the planar double pendulum. It is small enough that every term can be inspected. Reproduce the observed arm and club state at one instant, then calculate the full acceleration. Next set the selected arm or wrist moment to zero without changing the state and calculate again. The difference is the instantaneous influence of that intervention under the model. Repeat the comparison over the interval and integrate power if the question concerns work. This does not prove that a human chose the modeled control, but it turns "the arms gave energy to the club" into a calculation that can be checked.

The first negative control is equally important. Keep the measured motion but change the coordinate representation, or calculate the energy balance about a different convenient point, and verify that physical totals transform as they should. Refine the timestep and solver tolerance. Reverse a moment arm whose sign is known. Remove gravity. If an alleged transfer appears only at one timestep or changes sign under an equivalent coordinate convention, the result is an implementation artifact rather than a robust mechanism.

Suppose the double-pendulum result survives. The claim has earned one rung, not the entire ladder. Add the two-hand constraint model. The club now receives a resultant force, a force-generated couple from the separation of the hands, and any directly represented grip moment. Ask whether the late negative couple is still present and whether it closes the club's moment balance. Collapse the hand points together as a geometric negative control. The force-generated couple must vanish when its lever arm vanishes. If it does not, either the calculation or the interpretation is wrong.

Next replace the rigid club with a flexible shaft. Some power can now enter strain energy rather than immediate rigid-body motion. A negative handle moment might reduce club angular speed in one short interval while loading the shaft for a later return. It might instead remove energy that damping never returns. Only a strain-energy and dissipation ledger can distinguish those stories. The casual phrase "the shaft kicked" becomes several measurable questions: when did the shaft store energy, how much returned before impact, in which bending or torsional mode, and how sensitive was the result to stiffness and damping?

Then let the base move. Prescribing the shoulder path can hide the reaction loads and work needed to create it. A moving-base model asks whether the same distal effect survives when the torso and support are allowed to respond. Finally move into three dimensions, where a moment that looks negative about one planar axis may coexist with positive work about another axis. At every rung, the reviewer asks the same question: did the mechanism survive, change meaning, or disappear when a previously hidden degree of freedom became available?

Human evidence enters after the model has produced distinct predictions. If two candidate strategies generate nearly identical club motion but different bilateral grip couples, measure the grip wrenches. If they predict different shaft histories, measure strain. If they predict a directionally persistent arm channel and a reversing wrist channel, estimate joint moments with the internal hand wrench constrained rather than guessed. EMG can help establish activation timing, but it cannot by itself supply the missing hand force or joint moment. The measurement package should be chosen because it separates the explanations, not because the instrument happens to be available.

At this point the original sentence can be rewritten carefully: "During this interval, the measured hand point slowed in the laboratory frame while the clubhead accelerated. In the declared linked-segment model, arm-club constraint reactions and the measured net grip wrench produced positive power on the club, while the modeled arm system lost kinetic energy. The energy balance closed within the stated residual, and the result survived the listed timestep, coordinate, and model-fidelity checks." That sentence is longer than the slogan, but every phrase earns a specific piece of evidence.

The conclusion can still fail. The measured internal wrench may contradict the inverse-dynamics allocation. Shaft strain may show that energy returned after the useful impact window. A participant may create the same outcome with a different sequence. The result may vanish when shoulder motion is no longer prescribed. Those are not embarrassments. They identify exactly where a simple story stopped working and where a better one must begin.

30.2.18 A Full Conversation About "Keeping the System Loaded"

Consider a second familiar claim: "Keep tension in the system through the transition so there is no slack." The curious golfer may be describing a very real sensation of connectedness. Perhaps grip pressure remains steady, the arms feel organized against the torso, and the club changes direction without a little uncontrolled wobble. That sensation is valuable as a report. It still contains several possible mechanisms.

The reviewer replaces "tension" with candidate measurements. Is there a nonzero resultant grip force, a nonzero differential hand couple, forearm muscle activation, scapular-muscle activation, shaft strain, tendon force, or high tangent stiffness about a joint? These variables can move independently. Two opposing muscle groups can raise stiffness while their net joint moment stays near zero. Two hand forces can create a large couple with a small resultant force. A shaft can remain strained while the hands briefly reduce their active moment. "Loaded" therefore needs both a location and a quantity.

Now compare the two preparation extremes developed earlier. In the first, arm-and-shoulder channels create more of the backswing push–pull tendency while the wrists resist and regulate the club. The broad channel directions persist into early downswing. In the second, wrist moments create more of the backswing turning tendency while the arms resist; around transition the arms take over as the positive driver and the wrists oppose release. The visible club path can be matched even though the internal allocation and history differ.

In the declared dead-zone model, persistent direction has an advantage because it does not cross the modeled unloaded zone. Complete role reversal must travel through that zone before torque rebuilds. This is a useful conditional result, not proof that a human wrist contains backlash. Set the dead zone to zero and the advantage largely disappears. That negative control tells us precisely what produced the simulated difference.

A biological system can preserve readiness during a net reversal without literal continuous torque. Antagonistic preactivation may keep short-range stiffness high. Tendon or shaft strain may preserve a loaded elastic state. Contact forces may remain nonzero while their moment about a selected axis changes sign. Conversely, a person may report continuous effort while the task-relevant force mode briefly approaches zero. Feel and mechanics may be related, but their mapping must be calibrated rather than assumed.

The two strategies also have costs that a peak-speed-only model will miss. Persistent co-contraction can raise joint compression, metabolic cost, fatigue, and sensitivity to unwanted high-frequency loading. A role reversal can use different muscle groups, exploit changing leverage, and permit useful reorientation. Some compliance can store energy, absorb disturbances, and protect tissue. Too much compliance or a true contact gap can delay force rise. There is no reason to expect one point on this tradeoff surface to be best for every participant and every objective.

A decisive experiment therefore matches the visible task while manipulating allocation. Measure bilateral grip wrenches, grip pressure, wrist and shaft kinematics, shaft strain, transition-aligned force rise, and suitable forearm and shoulder-girdle EMG. Estimate tangent stiffness with an ethical bounded perturbation or system-identification method. Compare not only clubhead speed but face and path variability, peak joint demand, repeatability, and response to a small disturbance. Hold participants out when selecting a strategy so the test does not turn a group average into a universal prescription.

Interpret the outcomes in stages. If the instructions do not change measured allocation, the manipulation failed. If allocation changes but the predicted gap or stiffness difference does not, the dead-zone explanation weakens. If the mediator changes but performance does not, the mechanism may be real yet unimportant for the task. If performance changes without the predicted mediator, another pathway deserves attention. If participants split into repeatable subgroups, the scientifically useful result may be a conditional map rather than a winner.

The careful practical translation is modest: a golfer and coach can distinguish continuous engagement from maximum effort and can ask which measurable mode a cue is intended to change. They should not infer that more tension is always better, that wrists should be passive, or that one scapular action uniquely creates the club couple. The framework earns better questions and experiments; it does not yet earn a universal body instruction.

30.2.19 How to Carry the Method Into a New Question

When a new swing explanation appears, first write the observation without a causal verb. Next define the system boundary, coordinate frame, interval, and quantity. Draw the smallest model that contains the proposed mechanism and a negative control capable of removing it. Record the state before applying the counterfactual so that history is not silently reset. Close force, moment, power, and energy balances at the level required by the claim.

Then name at least one serious alternative. Increase model fidelity only where that alternative exploits something the smaller model omitted. Derive predictions that differ in measurable variables, including an internal variable when external motion is not identifying. Specify the residual, sensitivity, and participant-held-out criteria before looking for a preferred result.

Finally, translate the result twice. The scientific translation states the model, intervention, observable, uncertainty, and falsifier. The ordinary translation uses a concrete picture and says where that picture breaks. If the two translations disagree, revise them until neither borrows certainty from the other. That repeated movement between intuition and audit is how an accessible explanation can remain genuinely scientific rather than merely simple.

30.2.20 Chapter Checkpoint

The curious golfer asks questions that matter. The skeptical reviewer makes their meanings stable enough to test. Together they replace simple slogans with a linked chain: define the system and quantity, choose the smallest adequate model, preserve state history, measure the predicted observables, run negative controls, compare alternatives, and declare what would refute the claim. That chain is the real framework of the project.

30.3 Where the Picture Breaks

A dialogue can make disagreements look cleaner than they are. Real experiments face noisy measurements, participant variation, coupled objectives, missing data, and ethical limits. Some questions cannot be settled by one study, and a null result can reflect weak manipulation or inadequate measurement rather than absence of a mechanism. The two voices also do not replace domain experts in biomechanics, motor control, instrumentation, statistics, coaching, or clinical care. They provide a disciplined entry point for those collaborations.

Go Deeper: Audit the claims against the complete scientific monograph, use the open reviewer workbench, and inspect the project-level falsification matrix.

Glossary

Acceleration: The rate at which velocity changes. It can change speed, direction, or both.

Ablation: A controlled model experiment in which one mechanism is removed or neutralized while the remaining conditions are held as comparable as possible.

Constraint: A rule limiting allowable motion, such as two linked segments sharing a joint or two hands remaining attached to one grip.

Coriolis Term: A cross-speed part of a motion equation under a declared coordinate convention. In the two-link example it requires simultaneous arm rotation and relative wrist motion; it is not a new external or muscle force.

Centripetal and Centrifugal: In an inertial description, centripetal names the inward requirement for curved motion. Centrifugal is the outward inertial interpretation in a rotating description. They are not two forces to add.

Counterfactual: A carefully stated "what if" comparison used to attribute a quantity or test whether a proposed mechanism persists.

Drift: The part of modeled acceleration produced by the current state and passive terms when the declared control input is set to zero at that instant.

Energy: A scalar accounting quantity for the capacity of a chosen system to produce change. Its numerical value depends on the system boundary and reference.

Falsifier: An observation or test result that would count against a claim.

Identifiability: Whether available observations can uniquely or usefully determine a model parameter, internal force, or mechanism.

Impulse: Force integrated over time. Angular impulse is moment integrated over time.

Interaction Force: A force transmitted at a connection between parts of a linked system.

Moment or Torque: The turning effect of a force or a direct rotational couple about a declared point and axis.

Power: The instantaneous rate of energy transfer or work, measured in watts.

Preload: A nonzero internal load already present before a command or event changes.

Robustness: Performance retained across a declared set of disturbances, uncertainties, participants, or model parameters.

State: The collection of variables needed by a model to predict its future from the present, commonly configuration, velocity, and stored internal states.

Squared-Speed Term: A velocity-dependent equation term containing one generalized speed squared. Its detailed allocation is coordinate-dependent.

Wrench: A combined force and moment expressed about a declared reference point and in a declared coordinate frame.

Work: Energy transferred by a force through displacement or by a moment through angular displacement.

Pointwise Zero-Torque Counterfactual (ZTCF) Force: The generalized-force representation predicted at a declared state when the selected control input is set to zero for a pointwise decomposition (the pointwise drift generalized force).

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